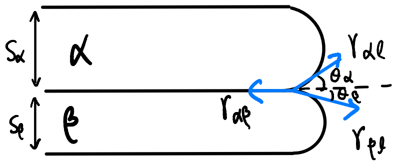
HW 6 화학과 손예림

1. (a)



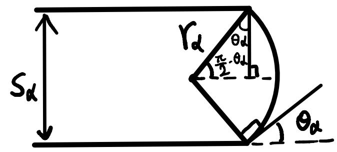
interface 에서 각 surface energy는 force balance를 이루게 된다.

$$\gamma_{\alpha\beta} = \gamma_{\alpha l}\cos\theta_\alpha + \gamma_{\beta l}\cos\theta_\beta$$

$$\gamma_{\alpha}\sin\theta_{\alpha l} = \gamma_{\beta l}\sin\theta_\beta$$

$\gamma_\alpha \neq \gamma_\beta$ 일 때 위 두 식을 만족하는 유의미한 $\theta_\alpha, \theta_\beta$는 $0<\theta_\alpha, \theta_\beta<90$ 이므로, 각 layer의 tip은 curvature를 가진다.

(b)



$S_\alpha = 2r_\alpha\cos\theta_\alpha$, 동일한 과정으로 $S_\beta = 2r_\beta\cos\theta_\beta$

(a)에서 $\gamma_{\alpha l}^2\sin^2\theta_\alpha = \gamma_{\beta l}^2\sin^2\theta_\beta \Rightarrow \gamma_{\alpha l}^2 - \gamma_{\alpha l}^2\cos^2\theta_\alpha = \gamma_{\beta l}^2 - \gamma_{\beta l}^2\cos^2\theta_\beta$

$$(\gamma_{\alpha\beta} - \gamma_{\alpha l}\cos\theta_\alpha)^2 = \gamma_{\beta l}^2\cos^2\theta_\beta = \gamma_{\beta l}^2 - \gamma_{\alpha l}^2 + \gamma_{\alpha l}^2\cos^2\theta_\alpha$$

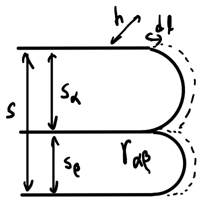
$$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha l}\cos\theta_\alpha + \gamma_{\alpha l}^2\cos^2\theta_\alpha = \gamma_{\beta l}^2 - \gamma_{\alpha l}^2 + \gamma_{\alpha l}^2\cos^2\theta_\alpha$$

$$\cos\theta_\alpha = \frac{\gamma_{\alpha\beta}^2 + \gamma_{\alpha l}^2 - \gamma_{\beta l}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha l}}$$

위와 동일하게, $\cos\theta_\beta = \dfrac{\gamma_{\alpha\beta}^2 - \gamma_{\alpha l}^2 + \gamma_{\beta l}^2}{2\gamma_{\alpha\beta}\gamma_{\beta l}}$

$\therefore\ r_\alpha = \dfrac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha l}}{(\gamma_{\alpha\beta}^2 + \gamma_{\alpha l}^2 - \gamma_{\beta l}^2)}$, $r_\beta = \dfrac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta l}}{(\gamma_{\alpha\beta}^2 - \gamma_{\alpha l}^2 + \gamma_{\beta l}^2)}$

(c)



$$\Delta G_{capillary} = \Sigma\gamma_i\frac{dA_i}{dn_i} = \Sigma\gamma_i\frac{h\,dl}{(S_i h\,dl/V_{m,i})} = \Sigma\gamma_i\frac{V_{m,i}}{S_i}$$

$$= \gamma_{\alpha\beta}\frac{V_{m,\alpha}}{S_\alpha} + \gamma_{\alpha\beta}\frac{V_{m,\beta}}{S_\beta}$$

$V_{m,\alpha} = \dfrac{S_\alpha}{S}V_m^L$, $V_{m,\beta} = \dfrac{S_\beta}{S}V_m^L$ 일 때,

$$\therefore\ \Delta G_{capillary} = \frac{2\gamma_{\alpha\beta}}{S}V_m^L$$

이는 α/β interfacial energy를 고려하여 얻어진 free energy 증가와 같다.