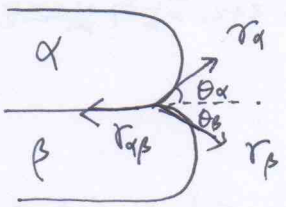
Problem #6 20202238 구관리과

1)



Force balance에 의해,

$$\gamma_{\alpha\beta} = \gamma_\alpha \cos\theta_\alpha + \gamma_\beta \cos\theta_\beta \quad ①$$

$$\gamma_\alpha \sin\theta_\alpha = \gamma_\beta \sin\theta_\beta \quad ②$$

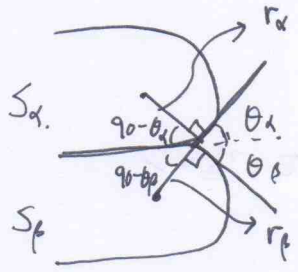
curvature가 없는 경우 $\theta_\alpha = \theta_\beta = 90°$ 가 된다.

② 식에 의해 $\gamma_\alpha = \gamma_\beta$ 가 성립하고, ① 에 의해 $\gamma_{\alpha\beta} = 0$ 이 된다.

하지만 $\gamma_{\alpha\beta} = 0$ 이 되기는 $\alpha = \beta$ 인 경우가 아니라면 힘들고

lamellar structure의 tip에서는 curvature가 생긴다.

2)



$$r_\alpha \sin(90-\theta_\alpha) = \frac{S_\alpha}{2} \rightarrow r_\alpha = \frac{S_\alpha}{2}\frac{1}{\cos\theta_\alpha}$$

$$r_\beta \sin(90-\theta_\beta) = \frac{S_\beta}{2} \rightarrow r_\beta = \frac{S_\beta}{2}\frac{1}{\cos\theta_\beta}$$

위의 ①, ② 식에서

$$\gamma_{\alpha\beta} = \gamma_\alpha \sqrt{1-\sin\theta_\alpha^2} + \gamma_\beta \cos\theta_\beta \rightarrow \gamma_{\alpha\beta}^2 + \gamma_\beta^2\cos^2\theta_\beta - 2\gamma_{\alpha\beta}\gamma_\beta\cos\theta_\beta = \gamma_\alpha^2 - \gamma_\alpha^2\sin^2\theta_\alpha = \gamma_\beta^2\sin^2\theta_\beta \quad \because ②$$

이항 후 양변제곱. $\rightarrow \gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2 = 2\gamma_{\alpha\beta}\gamma_\beta\cos\theta_\beta$

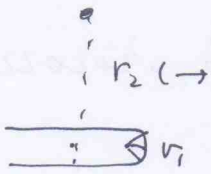
$$\therefore \cos\theta_\beta = \frac{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}{2\gamma_{\alpha\beta}\gamma_\beta}$$

$$\cos\theta_\alpha = \frac{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}{2\gamma_{\alpha\beta}\gamma_\alpha}$$ (위와 같은 방법)

$$\therefore r_\alpha = \frac{S_\alpha}{2}\frac{1}{\cos\theta_\alpha} = \frac{S_\alpha \gamma_{\alpha\beta}\gamma_\alpha}{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}$$

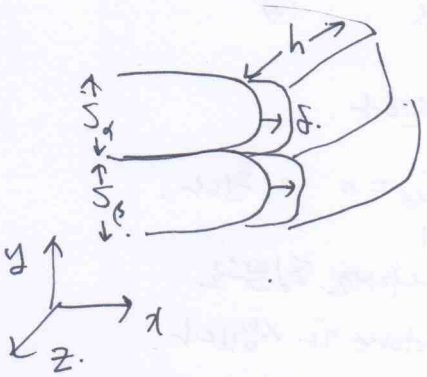
$$r_\beta = \frac{S_\beta}{2}\frac{1}{\cos\theta_\beta} = \frac{S_\beta \gamma_{\alpha\beta}\gamma_\beta}{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}$$

3) Capillary effect: $\Delta P = \gamma\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$



lamellar 구조에서는 $r_1 = r_1$, $r_2 \to \infty$ 라고 할 수 있으므로

$\Delta P = \frac{\gamma}{r_1}$, $\Delta G = \Delta P \cdot V_m = \frac{\gamma}{r_1} V_m$.



계면 방향으로 길이 δ 만큼 증가한다면, $V_m = Sh\delta$ 만큼 증가한다.

$\therefore \Delta G_{capillary} = \frac{\gamma_\alpha}{r_\alpha} S_\alpha h\delta + \frac{\gamma_\beta}{r_\beta} S_\beta h\delta$.

$$= \left(\frac{\gamma_{\alpha\beta}^2 + \gamma_\alpha^2 - \gamma_\beta^2}{\gamma_{\alpha\beta}} + \frac{\gamma_{\alpha\beta}^2 + \gamma_\beta^2 - \gamma_\alpha^2}{\gamma_{\alpha\beta}} \right) \cdot h\delta$$

$$= 2\gamma_{\alpha\beta} h\delta$$

$\Delta G_{IF} = \frac{2\gamma_{\alpha\beta}}{S} V_m = 2\gamma_{\alpha\beta} h\delta$ 으로 같다는 것을 확인할 수 있다.