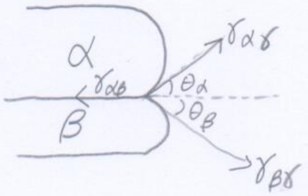
Phase transformation #6　　　20192742 손수정

1. (a) Considering surface tension,



$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta$$

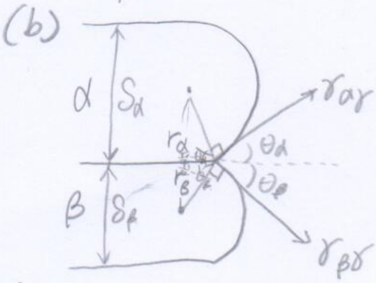
이 때 curvature가 없다면 $\theta_\alpha = \theta_\beta = 90°$

$$\cos\theta_\alpha = \cos\theta_\beta = 0$$

$\gamma_{\alpha\beta} = 0$ 이므로 상식적이지 않다.

즉 $\gamma_{\alpha\beta} \neq 0$ 이라면 curvature는 존재해야 한다

(b)



$$S_\alpha = 2r_\alpha\sin\left(\frac{\pi}{2}-\theta_\alpha\right) = 2r_\alpha\cos\theta_\alpha \qquad S_\beta = 2r_\beta\sin\left(\frac{\pi}{2}-\theta_\beta\right) = 2r_\beta\cos\theta_\beta$$

$$\gamma_{\alpha\gamma}\sin\theta_\alpha = \gamma_{\beta\gamma}\sin\theta_\beta \qquad \gamma_{\alpha\gamma}^2\sin^2\theta_\alpha = \gamma_{\beta\gamma}^2\sin^2\theta_\beta \qquad \gamma_{\alpha\gamma}^2(1-\cos^2\theta_\alpha) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_\beta)$$

$$\cos\theta_\beta = \sqrt{1-\frac{\gamma_{\alpha\gamma}^2}{\gamma_{\beta\gamma}^2}(1-\cos^2\theta_\alpha)}$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta = \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\sqrt{1-\frac{\gamma_{\alpha\gamma}^2}{\gamma_{\beta\gamma}^2}(1-\cos^2\theta_\alpha)}$$

$$\gamma_{\alpha\beta} - \gamma_{\alpha\gamma}\cos\theta_\alpha = \gamma_{\beta\gamma}\sqrt{1-\frac{\gamma_{\alpha\gamma}^2}{\gamma_{\beta\gamma}^2}(1-\cos^2\theta_\alpha)}$$

$$\gamma_{\alpha\beta}^2 - 2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}\cos\theta_\alpha + \cancel{\gamma_{\alpha\gamma}^2\cos^2\theta_\alpha} = \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2(1-\cancel{\cos^2\theta_\alpha})$$

$$\cos\theta_\alpha \fallingdotseq \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}$$

$$r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha} = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2} \qquad r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

(c) Capillary effect로 인한 $\Delta G = \frac{\gamma_{\alpha\gamma}}{r_\alpha} V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta} V_\beta$

$V_\alpha = \frac{S_\alpha}{S} V_m^\gamma \quad V_\beta = \frac{S_\beta}{S} V_m^\gamma$ 으로 생각하면

$$\Delta G = \left(\frac{S_\alpha \gamma_{\alpha\gamma}}{r_\alpha} + \frac{S_\beta \gamma_{\beta\gamma}}{r_\beta} \right) \frac{V_m^\gamma}{S}$$

$$= \left\{ \frac{\cancel{S_\alpha} \cancel{\gamma_{\alpha\gamma}} (\gamma_{\alpha\beta}^2 - \cancel{\gamma_{\beta\gamma}^2} + \cancel{\gamma_{\alpha\gamma}^2})}{\cancel{S_\alpha} \cancel{\gamma_{\alpha\beta}} \cancel{\gamma_{\alpha\gamma}}} + \frac{\cancel{S_\beta} \cancel{\gamma_{\beta\gamma}} (\gamma_{\alpha\beta}^2 - \cancel{\gamma_{\alpha\gamma}^2} + \cancel{\gamma_{\beta\gamma}^2})}{\cancel{S_\beta} \cancel{\gamma_{\alpha\beta}} \cancel{\gamma_{\beta\gamma}}} \right\} \frac{V_m^\gamma}{S}$$

$$= \frac{2\gamma_{\alpha\beta} V_m^\gamma}{S} = \Delta G_{IF}.$$