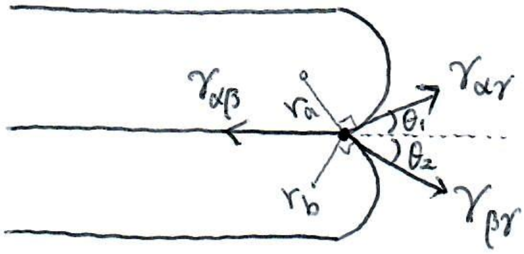
1. (a)



Interface 사이의 force balance

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\cos\theta_2$$

$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2$$

만약 layer tip 이 curvature를 가지지 않는다면 ($\theta_1 = \theta_2 = 90°$)

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cdot 0 + \gamma_{\beta\gamma}\cdot 0 = 0 \Rightarrow \text{unrealistic}.$$

∴ layer tip 은 curvature를 가져야 한다.

(b)

$$S_\alpha = 2r_a\cos\theta_1 \qquad S_\beta = 2r_b\cos\theta_2$$

$$\gamma_{\alpha\gamma}\sin\theta_1 = \gamma_{\beta\gamma}\sin\theta_2 \Rightarrow \gamma_{\alpha\gamma}^2\sin^2\theta_1 = \gamma_{\beta\gamma}^2\sin^2\theta_2.$$

$$\Rightarrow \gamma_{\alpha\gamma}^2(1-\cos^2\theta_1) = \gamma_{\beta\gamma}^2(1-\cos^2\theta_2)$$

$$\Rightarrow \cos\theta_2 = \sqrt{\frac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_1}{\gamma_{\beta\gamma}^2}}$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\cos\theta_1 + \gamma_{\beta\gamma}\times\sqrt{\frac{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\cos^2\theta_1}{\gamma_{\beta\gamma}^2}}$$

$$\gamma_{\alpha\beta} = \gamma_{\alpha\gamma}\frac{S_\alpha}{2r_a} + \sqrt{\gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_a^2}}$$

$$\Rightarrow \gamma_{\beta\gamma}^2 - \gamma_{\alpha\gamma}^2 + \cancel{\gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_a^2}} = \gamma_{\alpha\beta}^2 + \cancel{\gamma_{\alpha\gamma}^2\frac{S_\alpha^2}{4r_a}} - 2\gamma_{\alpha\beta}\gamma_{\alpha\gamma}\frac{S_a}{2r_a}$$

$$\Rightarrow \frac{1}{r_a} = \frac{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}} \Rightarrow r_a = \frac{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2}$$

$$r_\beta = \frac{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}{\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2}$$

(C) cylinder $\Rightarrow$ capillarity effect $\frac{\gamma}{r}V_m$

$$\Delta G = \frac{\gamma_{\alpha\gamma}}{r_a}V_\alpha + \frac{\gamma_{\beta\gamma}}{r_\beta}V_\beta$$

$$= \frac{\gamma_{\alpha\gamma}(\gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2 + \gamma_{\beta\gamma}^2)V_\alpha}{S_\alpha\gamma_{\alpha\beta}\gamma_{\alpha\gamma}} + \frac{\gamma_{\beta\gamma}(\gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2 + \gamma_{\alpha\gamma}^2)V_\beta}{S_\beta\gamma_{\alpha\beta}\gamma_{\beta\gamma}}$$

$\frac{V_\alpha}{S_\alpha} = \frac{V_\beta}{S_\beta} = \frac{V_m}{S}$ 가정한다면.

$$\Delta G = \frac{2\gamma_{\alpha\beta}V_m}{S} = \Delta G_{IF}(S)$$