

1. nucleation $\text{liquid} \rightarrow \text{liquid}$

$$dG = -SdT + VdP + \sum_i \mu_i dn_i + \sigma dA$$

$\downarrow T, P \text{ const.}$

$$dG = \cancel{\mu_{\text{vap}} dn_{\text{vap}}} + \mu_{\text{liq}} dn_{\text{liq}} + \sigma dA$$

$$= -(\mu_{\text{vap}} - \mu_{\text{liq}}) dn_{\text{liq}} + \sigma dA$$

$$) + (\mu_{\text{vap}} - \mu_{\text{liq}}) = \Delta G_{\text{vap}} \quad (\text{per volume of liquid})$$

$$\Delta G = -\Delta G_{\text{vap}} \cdot \left(\frac{4}{3}\pi r^3\right) + 4\pi r^2 \sigma$$

V_m : ~~molar volume~~ atomic volume

n : number of atoms in cluster

$$\frac{4}{3}\pi r^3 = n \cdot V_m, \quad r^3 = \frac{3nV_m}{4\pi}, \quad r = \left(\frac{3nV_m}{4\pi}\right)^{\frac{1}{3}}$$

$$4\pi r^2 = 4\pi \left(\frac{3nV_m}{4\pi}\right)^{\frac{2}{3}} = 4^{\frac{1}{3}} 9^{\frac{1}{3}} \pi^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V_m^{\frac{2}{3}} = (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V_m^{\frac{2}{3}}$$

$$\therefore \Delta G = -\Delta G_{\text{vap}} \cdot (nV_m) + (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V_m^{\frac{2}{3}} \cdot \sigma$$

2.

$$(a) \Delta G_n = -nV \Delta G_v + (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V^{\frac{2}{3}} \sigma$$

$$(b) \frac{d\Delta G}{dn} = 0 \Rightarrow \frac{d\Delta G}{dn} \bigg|_{n^*} = -V \Delta G_v + (36\pi)^{\frac{1}{3}} \cdot \frac{2}{3} n^{-\frac{1}{3}} \cdot V^{\frac{2}{3}} \sigma = 0$$

$$\Rightarrow n^* = \left(\frac{(36\pi)^{\frac{1}{3}} \cdot 2 \cdot V^{\frac{2}{3}} \cdot \sigma}{3V \Delta G_v} \right)^3 = \frac{36\pi}{3} \cdot \frac{2^3}{V} \left(\frac{\sigma}{\Delta G_v} \right)^3$$

$$= \frac{32}{3} \frac{\pi}{V} \left(\frac{\sigma}{\Delta G_v} \right)^3$$

(c)

$$\Delta G_{\text{dia}} = -n(G_{\text{vap}} - G_{\text{dia}}) + (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V_{\text{dia}}^{\frac{2}{3}} \sigma_{\text{dia}}$$

$$\Delta G_{\text{gr}} = -n \cancel{G_{\text{vap}}} (G_{\text{vap}} - G_{\text{gr}}) + (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} \cdot V_{\text{gr}}^{\frac{2}{3}} \cdot \sigma_{\text{gr}}$$

$$\Delta G_{\text{dia}} = \Delta G_{\text{gr}}$$

$$n(G_{\text{dia}} - G_{\text{gr}}) = (36\pi)^{\frac{1}{3}} \cdot n^{\frac{2}{3}} (V_{\text{gr}}^{\frac{2}{3}} \sigma_{\text{gr}} - V_{\text{dia}}^{\frac{2}{3}} \sigma_{\text{dia}})$$

$$n = \left[\frac{(36\pi)^{\frac{1}{3}} \cdot (V_{\text{gr}}^{\frac{2}{3}} \sigma_{\text{gr}} - V_{\text{dia}}^{\frac{2}{3}} \sigma_{\text{dia}})}{(G_{\text{dia}} - G_{\text{gr}})} \right]^3$$

if $\sigma_{\text{dia}} = 3.6$

$$n = \left(\frac{(36\pi)^{\frac{1}{3}} \cdot (8 \times 10^{-30})^{\frac{2}{3}} \times 3.1 - (6 \times 10^{-30})^{\frac{2}{3}} \times 3.6}{0.02 \times 1.602 \times 10^{-19}} \right)^3 \approx 464$$

if $\sigma_{\text{dia}} = 3.65$

$$n \approx 145$$

if $\sigma_{\text{dia}} = 3.10$

$$n \approx 21$$

d)

$$\Delta G_{d'n} < \Delta G_{gr}$$

$$\Rightarrow n < \left[\frac{(36\pi)^{\frac{1}{3}} \cdot (V_{gr}^{\frac{2}{3}} \cdot \sigma_{gr} - V_{d'n}^{\frac{2}{3}} \cdot \sigma_{d'n})}{(G_{d'n} - G_{gr})} \right]^3$$

e)

$$n^* = \frac{32\pi}{3V_{gr}} \left(\frac{\sigma_{gr}}{\Delta G_{v,gr}} \right)^3 \Rightarrow \Delta G_{v,gr} = \left(\frac{32\pi}{3V_{gr} \cdot n^*} \right)^{\frac{1}{3}} \cdot \sigma_{gr}$$

$$= \left(\frac{32\pi}{3 \cdot (8 \times 10^{-30}) \cdot 100} \right)^{\frac{1}{3}} \cdot 3.1$$

$$= 1.071 \times 10^{10} \text{ J/m}^3$$

$$f) \frac{I_{gr}}{I_{d'n}} = \frac{A \exp\left(-\frac{\Delta G_{v,gr}^*}{kT}\right)}{A \exp\left(-\frac{\Delta G_{v,d'n}^*}{kT}\right)} = \exp\left(-\frac{\Delta G_{v,gr}^*}{kT} + \frac{\Delta G_{v,d'n}^*}{kT}\right)$$

$$= \exp\left(\frac{-\frac{16}{3}\pi \frac{\sigma_{gr}^3}{\Delta G_{v,gr}} + \frac{16}{3}\pi \frac{\sigma_{d'n}^3}{\Delta G_{v,d'n}}}{kT}\right)$$

$$= \exp\left(\frac{16}{3} \frac{\pi}{kT} \left(-\frac{\sigma_{gr}^3}{\Delta G_{v,gr}} + \frac{\sigma_{d'n}^3}{\Delta G_{v,d'n}}\right)\right)$$

$$\bullet \Delta G_{v,gr} = 1.071 \times 10^{10} \text{ J/m}^3$$

$$\bullet G_{d'n} - G_{gr} = V_{gr} \cdot \Delta G_{v,gr} - V_{d'n} \cdot \Delta G_{v,d'n}$$

$$\Delta G_{v,d'n} = \frac{V_{gr} \cdot \Delta G_{v,gr} - (G_{d'n} - G_{gr})}{V_{d'n}} = \frac{(8 \times 10^{-30}) \cdot (1.071 \times 10^{10}) - (0.02 \times 1.607 \times 10^{-14})}{6 \times 10^{-30}}$$

$$= 1.38 \times 10^{10} \text{ J/m}^3$$

$$\bullet \text{ If } T = 298\text{K}, \sigma_{d'n} = 3.6$$

$$\frac{I_{gr}}{I_{d'n}} = \exp\left(\frac{16\pi}{3 \cdot 1.38 \times 10^{-23}} \left(-\frac{3.1^3}{(1.071 \times 10^{10})^2} + \frac{3.6^3}{(1.38 \times 10^{10})^2}\right)\right)$$

$$= 1.103 \times 10^{-21}$$

$$\text{If } T = 298\text{K}, \sigma_{d'n} = 3.65$$

$$\frac{I_{gr}}{I_{d'n}} = 2.214 \times 10^{-3}$$

$$\text{If } T = 298\text{K}, \sigma_{d'n} = 3.7$$

$$\frac{I_{gr}}{I_{d'n}} = 1.514 \times 10^{16}$$

g) bulk 상태에서 graphite 가 diamond 보다 안정하지만 nano size 에서는 surface 에 의한 영향도 많이 받아 diamond 가 더 잘 형성될 수 있다.

$\sigma_{d'n}$ 가 ~~작을수록~~ diamond 가 더 잘 형성된다.
작을수록

- h) $\text{CH}_4 \rightarrow \text{C} + \text{H}_2$ 반응이 일어나면 C의 분압이 증가한다. 이 때 graphite가 형성되는 것은 critical number of atom of graphite 를 클렸다는 것이다.
- 이를 줄이기 위해서는 $\Delta G_{r,gr}$, driving force 가 증가했음을 알 수 있다.
- 즉, driving force of graphite 는 C 분압과 연관 있음을 알 수 있다.