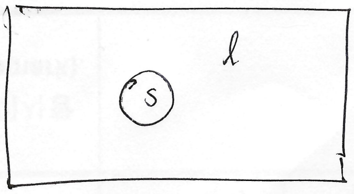
Phase Transformation HW #4 20202966 이정완.

1.



Free energy of transformation ΔG_v

$$\Delta G = -\frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma_{ls}$$

(γ_{ls} : liquid-solid interfacial energy for tin)

For r^*, critical radius, $\frac{\partial \Delta G}{\partial r}$ becomes 0.

$$-4\pi r^{*2} \Delta G_v + 8\pi r^* \gamma_{ls} = 0.$$

$$\therefore r^* = \frac{2\gamma_{ls}}{\Delta G_v}$$

$$\Delta G^* = -\frac{4}{3}\pi \cdot \frac{8\gamma_{ls}^3}{(\Delta G_v)^3} \Delta G_v + 4\pi\left(\frac{2\gamma_{ls}}{\Delta G_v}\right)^2 \gamma_{ls} = \frac{16\pi \gamma_{ls}^3}{3(\Delta G_v)^2}$$

Considering nucleation rate I,

$$I = I_o \exp\left(-\frac{\Delta G^*}{kT}\right) = f_o N_o \exp\left(-\frac{\Delta G^*}{kT}\right)$$

$$\ln I = \ln I_o - \frac{\Delta G^*}{kT}$$

The slope of $\ln I$ vs. $\frac{1}{T}$ is $-\frac{\Delta G^*}{k}$ which is -23.8×10^3 K.

$$\therefore \Delta G^* = +23.8 \times 10^3 K \cdot 1.38 \cdot 10^{-23} J/K = 32.8 \times 10^{-20} J = 3.28 \times 10^{-19} J$$

(a) $$\gamma_{ls} = \sqrt[3]{\frac{3(\Delta G_v)^2 \cdot \Delta G^*}{16\pi}} = \sqrt[3]{\frac{3 \cdot (-10^8 J \cdot m^{-3})^2 \cdot 3.28 \cdot 10^{-19} J}{16\pi}} = \sqrt[3]{0.1957 \cdot 10^{-3} J^3 m^{-6}}$$

$$= 0.581 \cdot 10^{-1} J \cdot m^{-2} = 0.0581 \ J \cdot m^{-2}$$

(b) $$r^* = \left|\frac{2\gamma_{ls}}{\Delta G_v}\right| = \left|\frac{2 \cdot 0.0581 J \cdot m^{-2}}{-10^8 J \cdot m^{-3}}\right| = 1.16 \times 10^{-9} m.$$

(c) The number of molecules (atoms) in cluster, which is denoted as n, and molecular volume v can be expressed :

$$\Delta G = -nv\Delta G_v + (36\pi)^{\frac{1}{3}} n^{\frac{2}{3}} v^{\frac{2}{3}} \gamma_{ls}$$

When $r = r^*$, $v = \frac{4}{3}\pi (r^*)^3$.

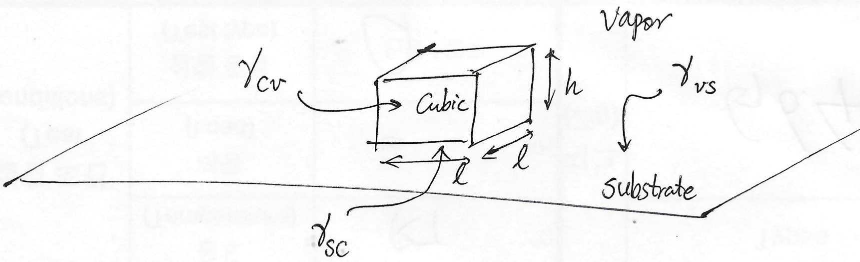
$$\left.\frac{\partial \Delta G}{\partial n}\right)_{n^*} = -\frac{4}{3}\pi (r^*)^3 \Delta G_v + \frac{2}{3} \cdot (36\pi)^{\frac{1}{3}} (n^*)^{-\frac{1}{3}} \cdot \left(\frac{4}{3}\pi (r^*)^3\right)^{\frac{2}{3}} \cdot 0.0581 J \cdot m^{-2} = 0.$$

$$(n^*)^{-\frac{1}{3}} = \frac{\frac{4}{3}\pi \ 3.375 \cdot 10^{-30} \cancel{m^3} \cdot 10^8 \cancel{J} \cdot \cancel{m^{-3}}}{\frac{2}{3} \cdot 4.836 \cdot 2.5986 \cdot 2.25 \cdot 10^{-20} \cancel{m^2} \cdot 0.0581 \cancel{J} \cdot \cancel{m^{-2}}} = 12.908 \times 10^{-2} = 0.129$$

$$\therefore (n^*)^{-1} = 2.15 \times 10^{-3}$$

$$n^* = 465$$

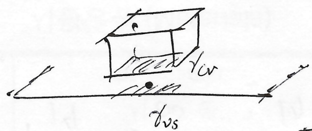
2.

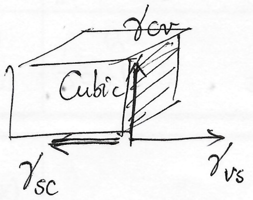


volume of cubic $V = l^2 h$.

When nucleation occurs, $\Delta G = -\Delta G_v \cdot l^2 h + \gamma \cdot (\text{Area})$

$$= -\Delta G_v\, l^2 h + \gamma_{cv} \cdot (4hl + l^2) + \gamma_{sc}\, l^2 - \gamma_{vs} \cdot l^2$$





For the balance, $\gamma_{sc} = \gamma_{vs}$.

$$\Delta G = -\Delta G_v\, l^2 h + \gamma_{cv}(4hl + l^2)$$

$$\left.\frac{\partial \Delta G}{\partial h}\right)_{h=h^*} = -\Delta G_v \underset{l^*}{\underset{\downarrow}{l^2}} + 4\gamma_{cv} \underset{l^*}{\underset{\downarrow}{l}} = 0 \qquad \therefore\ l^* = \frac{4\gamma_{cv}}{\Delta G_v}$$

$$\left.\frac{\partial \Delta G}{\partial l}\right)_{l=l^*} = -\Delta G_v \cdot 2l^* h^* + \gamma_{cv}(4h^* + 2l^*) = 0$$

$$h^*(4\gamma_{cv} - 2\cdot 4\gamma_{cv}) = -2\gamma_{cv} l^* = -\frac{8\gamma_{cv}^2}{\Delta G_v}$$

$$\therefore\ h^* = \frac{2\gamma_{cv}}{\Delta G_v}$$

$$\Delta G^* = -\Delta G_v \cdot (l^*)^2 h^* + \gamma_{cv}\left(4h^* l^* + (l^*)^2\right)$$

$$= -\Delta G_v \cdot \frac{16\gamma_{cv}^2}{(\Delta G_v)^2} \cdot \cancel{\frac{2\gamma_{cv}}{\Delta G_v}} + \gamma_{cv}\left\{\cancel{4 \cdot \frac{8\gamma_{cv}^2}{(\Delta G_v)^2}} + \frac{16\gamma_{cv}^2}{(\Delta G_v)^2}\right\}$$

$$= \frac{16\gamma_{cv}^3}{(\Delta G_v)^2}$$