HW#4 20182347 [illegible]

1. According to the theory of homogeneous nucleation,

$$\Delta G = -\frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma_{l-s}.$$

For a critical radius r, $\left.\frac{\partial \Delta G}{\partial r}\right|_{r^*} = -4\pi r^{*2}\Delta G_v + 8\pi r^* \gamma_{s-l} = 0.$

$$r^* = \frac{2\gamma_{s-l}}{\Delta G_v}, \quad \Delta G^* = \frac{16\pi\gamma_{s-l}^3}{3\Delta G_v^2}$$

On the other hand, nucleation rate $I = I_0 \exp\left(-\frac{\Delta G^*}{kT}\right)$ where $I_0 = f_0 N_0$

$\ln I = \ln I_0 - \frac{\Delta G^*}{kT}$. As the slope $-\frac{\Delta G^*}{k} = -23.8\times10^3\,K$,

$$\Delta G^* = 3.2844\times10^{-18}$$

(a) $\Delta G^* = \frac{16\pi\gamma_{s-l}^3}{3\Delta G_v^2} = \frac{16\times\pi\times\gamma_{s-l}^3}{3\cdot10^{16}} = 3.2844\times10^{-18}$. $\Rightarrow \gamma_{s-l}^3 = 1.96\times10^{-4}$.

$\therefore \gamma_{s-l} = 5.8\times10^{-2}\ J/m^2$.

(b) $r^* = \frac{2\times5.8\times10^{-2}}{10^8} = 1.16\times10^{-8}\ m$

(c) $\Delta G = -nV\Delta G_v + (36\pi)^{\frac{1}{3}} n^{\frac{2}{3}} V^{\frac{2}{3}} \gamma_{s-l}$

$$\left.\frac{\partial \Delta G}{\partial n}\right|_{n=n^*} = -\frac{4}{3}\pi r^{*3}\Delta G_v + \frac{2}{3}\cdot(36\pi)^{\frac{1}{3}}\cdot n^{-\frac{1}{3}}\cdot\left(\frac{4}{3}\pi r^{*3}\right)^{\frac{2}{3}}\cdot\gamma_{s-l}$$

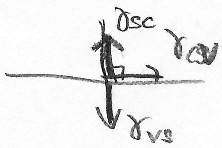
$$= -1.41\times10^{-28}\times10^8 + \frac{2}{3}\times4.84\times n^{-\frac{1}{3}}\times(1.41\times10^{-28})^{\frac{2}{3}}\times5.8\times10^{-2} = 0.$$

$\Rightarrow n^* = 465$개.

2. ΔG for nucleation $= -\Delta G_v\cdot V + \gamma\cdot A$

$$= -\Delta G_v\cdot l^2 h + \gamma_{cv}\cdot(l^2 + 4hl) + \gamma_{sc} l^2 - \gamma_{vs} l^2$$

For balance of interfacial energy,



$\Rightarrow \gamma_{sc} = \gamma_{vs}$.

$$\Rightarrow \Delta G = -\Delta G_v l^2 h + \gamma_{cv}\cdot(l^2 + 4hl).$$

For critical l^*, h^*,

$$\begin{cases} \left.\frac{\partial \Delta G}{\partial h}\right|_{h=h^*} = -\Delta G_v l^{*2} + 4l^*\gamma_{cv} = 0 \Rightarrow l^* = \frac{4\gamma_{cv}}{\Delta G_v} \\ \left.\frac{\partial \Delta G}{\partial l}\right|_{l=l^*} = -2\Delta G_v l^* h^* + \gamma_{cv}(2l^* + 4h^*) = 0 \Rightarrow h^* = \frac{2\gamma_{cv}}{\Delta G_v} \end{cases}$$

$$\Delta G^* \text{ for nucleation} = -\Delta G_v\cdot\left(\frac{4\gamma_{cv}}{\Delta G_v}\right)^2\left(\frac{2\gamma_{cv}}{\Delta G_v}\right) + \gamma_{cv}\left(\frac{16\gamma_{cv}^2}{\Delta G_v^2} + \frac{32\gamma_{cv}^2}{\Delta G_v^2}\right) = \frac{16\gamma_{cv}^3}{\Delta G_v^2}$$