HW#3 20212568 강승연

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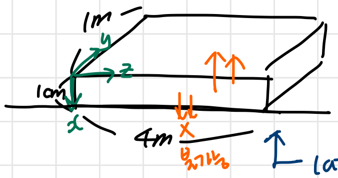


*측면적 infinity X

initial nitrogen = 100ppm

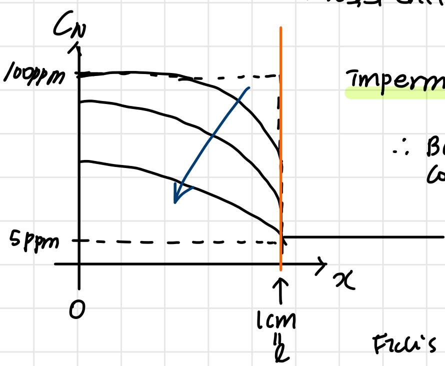
$T = 1000K$

$D_N = 4\times10^{-7}\ cm^2/s$

laid on the floor of a furnace heated to a T=1000K.

a) Neglect nitrogen losses from the bottom & edges of the plate. (y, z-direction)

→ x축의 diffusion만 생각!



Impermeable coating of one side

∴ Boundary condition

$$\begin{cases} C(0<x<1cm,\ t=0) = 100ppm \\ C(x \ge 1cm,\ t) = 5ppm \\ \dfrac{\partial C(x=0,t)}{\partial x} = 0 \end{cases}$$

Fick's second law ⇒ $\dfrac{\partial C(x,t)}{\partial t} = D\dfrac{\partial^2 C}{\partial x^2}$

b) $l = 1cm < 4\sqrt{Dt}$ (trigonal solution & 1^{st} term approximation) for simplification.

$\dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial x^2}$ using seperation of variables

$$C(x,t) = X(x)T(t) + 5$$

$$\frac{1}{DT}\cdot\frac{dT}{dt} = \frac{1}{X}\cdot\frac{d^2X}{dx^2} = -\lambda^2$$

$\dfrac{dT}{dt} + \lambda^2 DT = 0$; $T(x) = \exp[-\lambda_n^2 Dt]$

$\dfrac{d^2X}{dt^2} + \lambda^2 X = 0$; $X(x) = A\cdot\cos\lambda x + B\sin\lambda x$

$$\therefore C(x,t) = \sum_n (A\cdot\cos\lambda_n x + B\cdot\sin\lambda_n x)\exp[-\lambda_n^2 Dt]$$

⇒ with first term approximation

$$C(x,t) = (A\cos\lambda_n x + B\sin\lambda_n x)\exp[-\lambda_n^2 Dt]$$

Using boundary condition. $C(x=0) \neq 0$

$C(x=1\text{cm}, t) = 5\text{ppm}$ $\Big\} \Rightarrow B \rightarrow$ 유이정

$t=0 \Rightarrow C(x,t) = A\sin(\lambda_n x) + \cos(\lambda_n x) + 5 = 100$

$t \neq 0 \Rightarrow x = l$ 에서 $C(x,t) = T(t)\,(a_n \sin\lambda_n l + b_n \cos\lambda_n l) + 5 = 5$

$\Rightarrow a_n = 0$ ($\because$ cosine 개형이 나와야 함)

$$\lambda_n = \frac{n\pi + \frac{\pi}{2}}{l} = \frac{\pi}{l}\left(n+\frac{1}{2}\right) \text{ for the integer } n.$$

$$\therefore C(x,t) = A\cos\lambda x \cdot \exp[-\lambda^2 Dt] + 5\text{ppm}$$

Using Orthogonality Theorem

$t=0$ 일때,

$C(x,0) = T(0)\cdot A \cdot \cos\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right) + 5 = 100$, put $T(0)\cdot A = A'$

$$\Rightarrow A'\cos\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right) = 95$$

$$\Rightarrow A'\int_0^l \cos\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right)^2 dx = \int_0^l \cos\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right)\cdot 95 \cdot dx = \frac{A' \cdot l}{2}$$

$$A' = \frac{2}{l}\int_0^l 95 \cdot \cos\left(\frac{\pi}{l}n + \frac{1}{2}\right)x\,)dx$$

$$= \frac{2}{l}\cdot 95 \left[\frac{1}{(n+\frac{1}{2})\cdot\frac{\pi}{l}} \sin\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right)\right]_0^l$$

$$= \frac{2}{l}\cdot 95 \cdot \left(\frac{1}{(n+\frac{1}{2})\cdot\frac{\pi}{l}}\right) = \frac{2\cdot 95}{\pi(n+\frac{1}{2})}$$

$\rightarrow$ 그래프 개형에서 유추 ; $n =$ even number

$$C_n = \frac{190}{\pi(n+\frac{1}{2})}\cdot\cos\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)x\right)\cdot\exp\left[-\left(\frac{\pi}{l}\left(n+\frac{1}{2}\right)\right]^2 Dt\right] + 5$$

for even number n.

$$\therefore C(x,t) = \frac{4C_0}{\pi}\cos\left(\frac{\pi x}{2l}\right)\cdot\exp\left[-\frac{\pi^2 Dt}{4l^2}\right] + 5$$

With first-term approximation

$$\boxed{C = \frac{380}{\pi}\cos\left(\frac{\pi}{2l}x\right)\exp\left(-\left(\frac{\pi}{2l}\right)^2 Dt\right) + 5}$$

c) $C = \frac{C_0}{2}$ 가 되는 시간을 구해라.

$$\frac{C_0}{2} = \frac{4C_0}{\pi} \int_0^1 \cos\left(\frac{\pi x}{2}\right) \exp[-\pi^2 Dt]\, dx + 5 .$$ ($\because$ t is independent to x)

$$\frac{\pi}{8} - 5 = \int_0^1 \cos\left(\frac{\pi x}{2}\right) \exp[-\pi^2 Dt]\, dx = \frac{2}{\pi} \exp[-\pi^2 Dt]$$

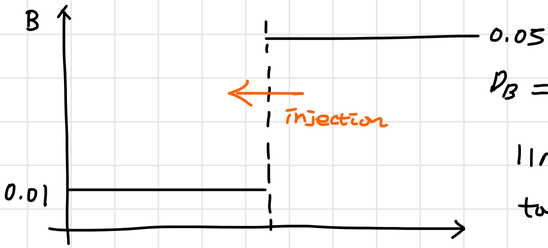
$\Rightarrow$ solution for this = 540,000 sec

d) half conc. will increase if D is dependent on concentration.

As time goes by the conc. decrease & so does for D.

Therefore, the diffusion amount will decrease compared to independent D.

#2)



$D_B = 4.329 \times 10^{-7}\, EXP[-147723(J)/RT]$

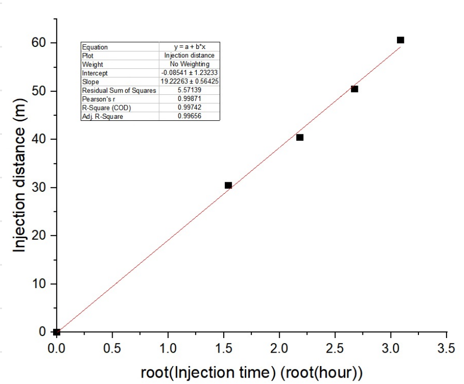
$1173K < T < 1473K$

target value $= 0.03$

$D_B = 1.195 \times 10^{-13}\ m^2/s = 1.195 \times 10^{-9}\ cm^2/s$

$1m = 10^2 cm$

a)

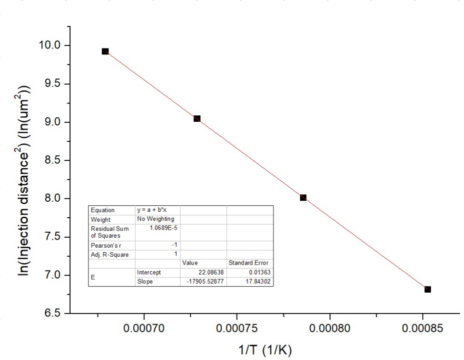


	A(X1)	B(X2)	C(Y2)
Long Name	Injection time	root(Injection time)	Injection distance
Units	hour	root(hour)	m
Comments			
F(x)=		A^(1/2)	
1	0	0	0
2	2.38	1.54272	30.5
3	4.77	2.18403	40.4
4	7.16	2.67582	50.51
5	9.54	3.08869	60.61

$\therefore l \propto \sqrt{t}$

b)

Temp.	D (cm²/sec)	Injection distance after 24hr (um)
1173K	1.19547×10^{-9}	30.3
1273K	3.92924×10^{-9}	55.11
1373K	1.08592×10^{-8}	92.18
1473K	2.61426×10^{-8}	143.29



$\ln(l^2) \propto \frac{1}{T}$ 의 관계가 성립.

c) $l \propto \sqrt{t}$, $\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$ ⇒ injection distance $\sqrt{Dt}$에 비례

$$l = a\sqrt{Dt}$$

$$\ln l = \ln a + \frac{1}{2}(\ln D + \ln t)$$

$$\begin{array}{rl} & \ln l_1 = \ln a + \frac{1}{2}(\ln D_1 + \ln t_1) \\ - & \ln l_2 = \ln a + \frac{1}{2}(\ln D_2 + \ln t_2) \\ \hline \end{array}$$

$$\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\left(\ln\left(\frac{D_1}{D_2}\right) + \ln\left(\frac{t_2}{t_2}\right)\right)$$

$$D = D_0 \exp\left(-\frac{Q}{RT}\right)$$ 대입

같은시간인 경우, $t_1 = t_2$

$$\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\ln\left(D_0 \exp\left(-\frac{Q}{RT_1}\right)\right) - \frac{1}{2}\ln\left(D_0 \exp\left(-\frac{Q}{RT_2}\right)\right)$$

$$= -\frac{Q}{2R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$T_1 = 1173K$, $T_2 = 1273K$ for $t = 2.4hr$

$l_1 = 30.3\mu m$, $l_2 = 55.6\mu m$

$$\ln\left(\frac{30.3}{55.6}\right) = -\frac{Q}{2\times 8.314}\left(\frac{1}{1173} - \frac{1}{1273}\right)$$

$$\therefore Q = 150{,}723\text{ J} \simeq 147{,}723\text{ J}$$