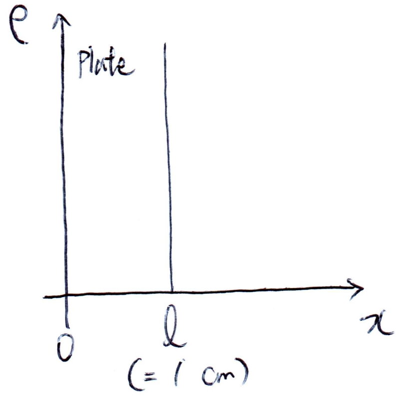
1. (a) $$\frac{\partial \rho}{\partial t} = -\frac{\partial J}{\partial x} = -\frac{\partial}{\partial x}\left(-D\frac{\partial \rho}{\partial x} + C \cdot V\right)$$

D가 일정한 상수이고 유동속도 (V)가 0인 경우

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2} \quad \begin{cases} \rho\,(0 \leq x < l,\ t=0) = 100 \text{ ppm} \\ \rho\,(x > l,\ t \geq 0) = 5 \text{ ppm} \end{cases}$$



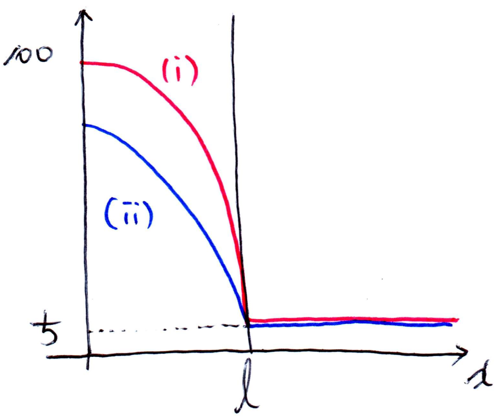
(b)

i) $l > 10\sqrt{Dt}$

$$\rho\,(x, t) = 95 \operatorname{erf}\left(\frac{l - x}{2\sqrt{Dt}}\right) + 5$$

ii) $l < 10\sqrt{Dt}$

$$\rho\,(x, t) = 95 \times \frac{4}{\pi}\sin\frac{\pi(x+l)}{2l} \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) + 5$$



(C) $\langle \rho \rangle = \dfrac{1}{l}\displaystyle\int_0^l \rho(x,t)\,dx$

① $t=0$, $0 \le x < l \Rightarrow \rho(x,0) = 100$

$$\langle \rho \rangle_0 = \int_0^1 100\,dx = 100$$

② 초기 농도보다 50% 감소된 시간이 논은 경우

$$l < 10\sqrt{Dt}$$

$$\langle \rho \rangle_t = \frac{1}{l}\int_0^l \rho(x,t)\,dx$$

$$= \frac{1}{l}\int_0^l \left\{ 95 \times \frac{4}{\pi} \times \sin\frac{\pi(x+l)}{2l} \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) + 5 \right\} dx$$

$$= \frac{1}{l}\left[\frac{380}{\pi} \times \left(-\frac{2l}{\pi}\cos\frac{\pi(x+l)}{2l}\right) \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) + 5x \right]_0^l$$

$$= \frac{1}{l}\left\{ \frac{380}{\pi} \times \left(-\frac{2l}{\pi}\cos\pi\right) \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) + 5l - \frac{380}{\pi} \times \left(-\frac{2l}{\pi}\cos\frac{\pi}{2}\right) \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) \right\}$$

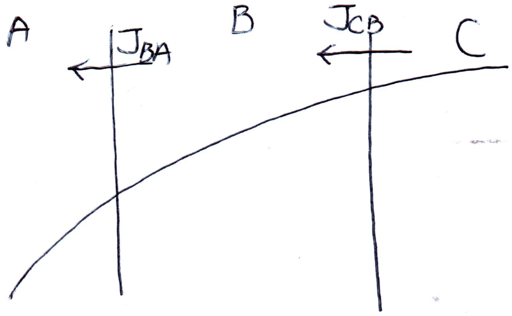
$$= \frac{1}{l} \times \frac{760\,l}{\pi^2} \times \exp\left(-\frac{\pi^2 Dt}{4l^2}\right) + 5l \quad (l = 1)$$

$$= \frac{760}{\pi^2}\exp\left(-\frac{\pi^2 Dt}{4}\right) + 5$$

$$\therefore \frac{\langle \rho \rangle_0}{2} = \langle \rho \rangle_t \Rightarrow 50 = \frac{760}{\pi^2}\exp\left(-\frac{\pi^2 Dt}{4}\right) + 5$$

$$t = -\frac{4}{\pi^2 \times 4.7 \times 10^{-6}} \ln\left(\frac{45\pi^2}{760}\right) = 65565\ \text{sec} = 18\ h$$

(d) 매우 작은 영역에 대한 농도가 아래와 같이 나타난다면



B 영역은 J_{CB} 보다 J_{AB} 가 크기 때문에 확산이 $C \rightarrow A$ 방향으로 일어난다.

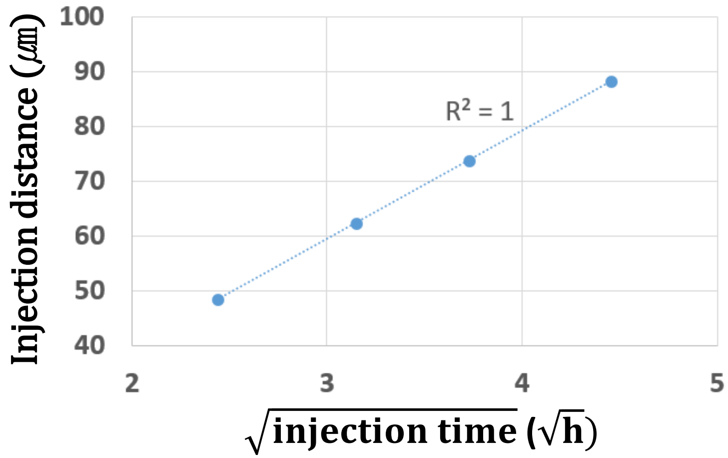
동일한 농도 profile 에서 농도가 증가할수록 diffusivity가 증가한다면 J_{CB} 와 J_{AB} 사이의 차이가 줄어들기 때문에 (C) 문제의 초기 농도보다 50% 낮은 농도까지 떨어지는데 걸리는 시간은 더 길어질 것으로 예상된다.

Homework #3 - FDM

$l = Injection\ distance\ (\mu m)$
$h = Injection\ time\ (h)$
$T = Injection\ temperature\ (K)$

2.(a)

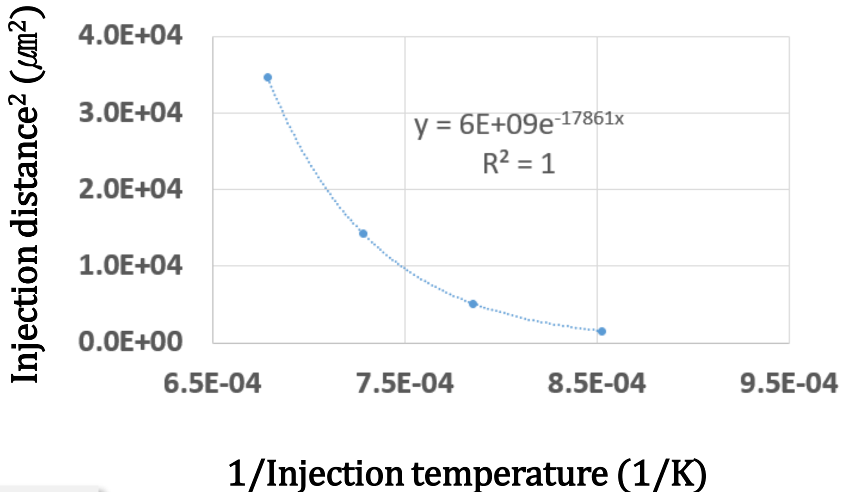
Injection time (h)	Injection distance (μm)
6.0	48.4
9.9	62.4
13.9	73.8
19.9	88.2



$$l \propto \sqrt{t}$$

2.(b)

Injection temperature (K)	Injection distance (μm)
1173	39.5
1273	71.7
1373	119.3
1473	186.4



$$l^2 \propto e^{1/T}$$

(C) $l \propto \sqrt{t}$, $l^2 \propto e^{1/T}$

$D = D_0 \exp\left(-\frac{Q}{RT}\right)$ 이기 때문에

$$l \propto \sqrt{Dt} \Rightarrow \frac{\sqrt{Dt}}{l} = \text{Const.}$$

$$\frac{\sqrt{D_1 t_1}}{l_1} = \frac{\sqrt{D_2 t_2}}{l_2} \Rightarrow \frac{D_0 \exp\left(-\frac{Q}{RT_1}\right) t_1}{l_1^2} = \frac{D_0 \exp\left(-\frac{Q}{RT_2}\right) t_2}{l_2^2}$$

$$-\frac{Q}{RT_1} + \ln t_1 - 2\ln l_1 = -\frac{Q}{RT_2} + \ln t_2 - 2\ln l_2$$

$$Q\left(\frac{1}{RT_2} - \frac{1}{RT_1}\right) = \ln\frac{t_2}{t_1} + 2\ln\frac{l_1}{l_2}$$

$$Q = \left(\ln\frac{t_2}{t_1} + 2\ln\frac{l_1}{l_2}\right)\left(\frac{RT_1T_2}{T_1 - T_2}\right)$$

FDM 데이터로부터 $\begin{cases} t_1 = 3.97 & l_1 = 39.4 & T_1 = 1173 \\ t_2 = 4.00 & l_2 = 71.7 & T_2 = 1273 \end{cases}$

대입해서 계산하면 $Q = 147725 \approx 147723$