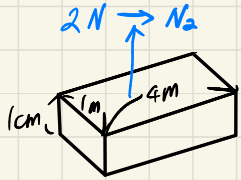
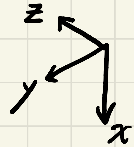
Phase transformation HW #3 20202852 김동현





1. Iron plate : 4m × 1cm × 1cm

$d_x = 1\,cm,\ d_y = 4\,m,\ d_z = 1\,m$

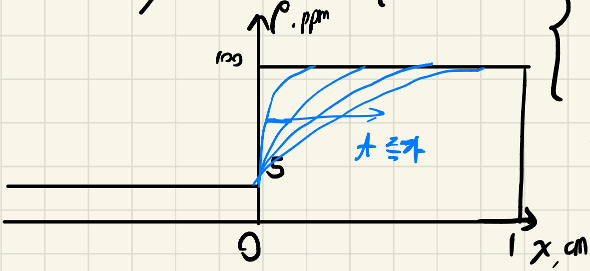
$T = 1000K,\quad D_N = 4\times10^{-7}\ cm^2/s$

(a) bottom, edge 로부터의 Nitrogen loss X.
& y, z 에 따른 Nitrogen potential 존재 X) ⇒ x축의 diffusion 만 고려 가능.

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot J = \nabla\cdot(D\nabla\rho) = D\nabla^2\rho = D\frac{\partial^2\rho}{\partial x^2}$$

$$J = -D\nabla\rho$$

Boundary Condition for $\rho(x,t)$:
$$\begin{cases} \rho(x>0,\ 0) = 100 \\ \rho(x\le 0,\ t) = 5 \\ \rho(0,t) = 5 \end{cases}$$
$0\le x<1\,cm,\quad t\ge 0$



(b) Semi-infinite source ⇒ $\rho_i = \dfrac{\rho_0}{\sqrt{4\pi Dt}}\exp\left(-\left(\dfrac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right)d\alpha$

지금은 둘 다 0이 아니므로

$$\begin{cases} \rho_0 = 100 & \text{for } a_i>0 \\ \rho_0 = 5 & \text{for } a_i<0 \end{cases} \Rightarrow \rho(x,t) = \sum\rho$$

$$= \int_{-\infty}^{0}\frac{5}{\sqrt{4\pi Dt}}\exp\left(-\left(\frac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right)d\alpha + \int_0^1\frac{100}{\sqrt{4\pi Dt}}\exp\left(-\left(\frac{x-\alpha_i}{\sqrt{4Dt}}\right)^2\right)d\alpha$$

let $y = \dfrac{x-\alpha}{\sqrt{4Dt}} \Rightarrow dy = -\dfrac{1}{\sqrt{4Dt}}d\alpha \Rightarrow d\alpha = -\sqrt{4Dt}\,dy$

$$\rho(x,t) = -\frac{5}{\sqrt{\pi}}\int_{\infty}^{\frac{x}{\sqrt{4Dt}}} e^{-y^2}\,dy - \frac{100}{\sqrt{\pi}}\int_{\frac{x}{\sqrt{4Dt}}}^{\frac{x-1}{\sqrt{4Dt}}} e^{-y^2}\,dy$$

$$= +\frac{5}{\sqrt{\pi}}\left(\int_0^{\infty} e^{-y^2}dy - \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-y^2}dy\right) + \frac{100}{\sqrt{\pi}}\left(\int_0^{\frac{x}{\sqrt{4Dt}}} e^{-y^2}dy - \int_0^{\frac{x-1}{\sqrt{4Dt}}} e^{-y^2}dy\right)$$

$$= +\frac{5}{2}\left(1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{4Dt}}} e^{-y^2}dy\right) + 50\left(\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{\sqrt{4Dt}}} e^{-y^2}dy - \frac{2}{\sqrt{\pi}}\int_0^{\frac{x-1}{\sqrt{4Dt}}} e^{-y^2}dy\right)$$

$$= +\frac{5}{2} - \frac{5}{2}\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50\operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)$$

(c) avg. nitrogen conc. $= \dfrac{\int_0^1 \rho(x,t)\,dx}{1} = \int_0^1 \rho(x,t)\,dx$

$$= \int_0^1 \left(-\frac{5}{2} + \frac{5}{2}\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50\operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)\right) dx$$

$$\int \operatorname{erf}(x)\,dx = x\operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}} + C$$

$x = \dfrac{y}{\sqrt{4Dt}}$ $\qquad dx = \dfrac{1}{\sqrt{4Dt}}dy$ $\qquad \dfrac{1}{\sqrt{4Dt}}\displaystyle\int \operatorname{erf}\left(\frac{y}{\sqrt{4Dt}}\right) dy$

$$= \frac{y}{\sqrt{4Dt}}\operatorname{erf}\left(\frac{y}{\sqrt{4Dt}}\right) + \frac{\exp\left(-\frac{y^2}{4Dt}\right)}{\sqrt{\pi}} + C$$

$$\therefore \int \operatorname{erf}\left(\frac{y}{\sqrt{4Dt}}\right) dy = y\operatorname{erf}\left(\frac{y}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{y^2}{4Dt}\right) + C$$

$$\begin{cases} \int \operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) dx = x\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{x^2}{4Dt}\right) + C \\ \int \operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right) dx = (x-1)\operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{(x-1)^2}{4Dt}\right) + C \end{cases}$$

$$\int_0^1 \rho(x,t)\,dx = \int_0^1 \left(+\frac{5}{2} - \frac{5}{2}\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + 50\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) - 50\operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right)\right)dx$$

$$= +\frac{5}{2} + \frac{95}{2}\left[x\operatorname{erf}\left(\frac{x}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{x^2}{4Dt}\right)\right]_0^1$$

$$-50\left[(x-1)\operatorname{erf}\left(\frac{x-1}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}\exp\left(-\frac{(x-1)^2}{4Dt}\right)\right]_0^1$$

$$= +\frac{5}{2} + \frac{95}{2}\left[\left(\operatorname{erf}\left(\frac{1}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}e^{-\frac{1}{4Dt}}\right) - \left(\sqrt{\frac{4Dt}{\pi}}\right)\right]$$

$$-50\left[\left(\sqrt{\frac{4Dt}{\pi}}\right) - \left(-\operatorname{erf}\left(\frac{-1}{\sqrt{4Dt}}\right) + \sqrt{\frac{4Dt}{\pi}}e^{-\frac{1}{4Dt}}\right)\right]$$

$$= +\frac{5}{2} - \frac{195}{2}\left(\sqrt{\frac{4Dt}{\pi}} - \sqrt{\frac{4Dt}{\pi}}e^{-\frac{1}{4Dt}}\right) + \frac{95}{2}\operatorname{erf}\left(\frac{1}{\sqrt{4Dt}}\right) - 50\operatorname{erf}\left(-\frac{1}{\sqrt{4Dt}}\right)$$

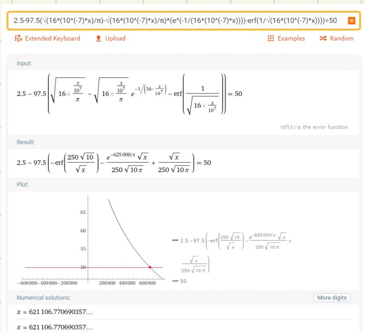
$$= +\frac{5}{2} - \frac{195}{2}\left(\sqrt{\frac{4Dt}{\pi}} - \sqrt{\frac{4Dt}{\pi}}e^{-\frac{1}{4Dt}} - \operatorname{erf}\left(\frac{1}{\sqrt{4Dt}}\right)\right)$$

$\operatorname{erf}(-x) = -\operatorname{erf}(x)$

$t=0$ 일 때 $\operatorname{erf}(\infty)=1 \Rightarrow \int_0^1 \rho(x,t)\,dx = 100$

$\int_0^1 \rho(x,t)\,dx = 50$ 인 t 값은 $t = 621106\,s \simeq 172.529\,h$ 이다.

$\therefore 172.529\,h.$



(d) 위의 사진처럼 t가 증가할수록 감소하는 함수다.

$y=\sqrt{Dt}$ 라 생각하자.

$D_1 > D_2$ 일 때 각각 절반에 도달하는 시간이 t_1, t_2 라고 하면

각 경우에서 $\sqrt{Dt}=y$ 값은 같아야 하므로 $D_1 t_1 = D_2 t_2$ 다.

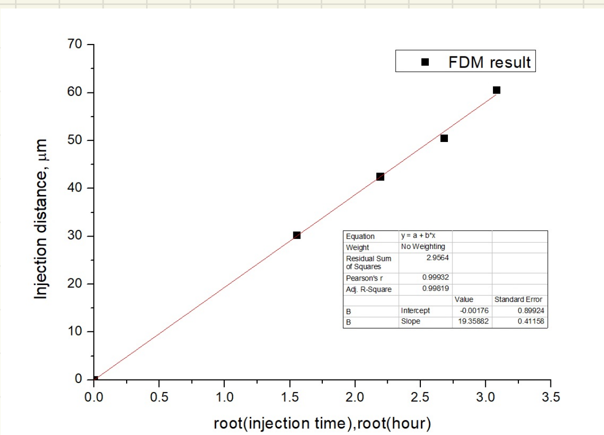
즉 $t_1 < t_2$ 가 성립하게 된다.

$\therefore$ D가 클수록 (C)의 답은 줄어들게 된다.

2. 1173 K 일 때 $D = 1.19547 \times 10^{-13}\ m^2/s = 1.19547 \times 10^{-9}\ cm^2/s$

(a)

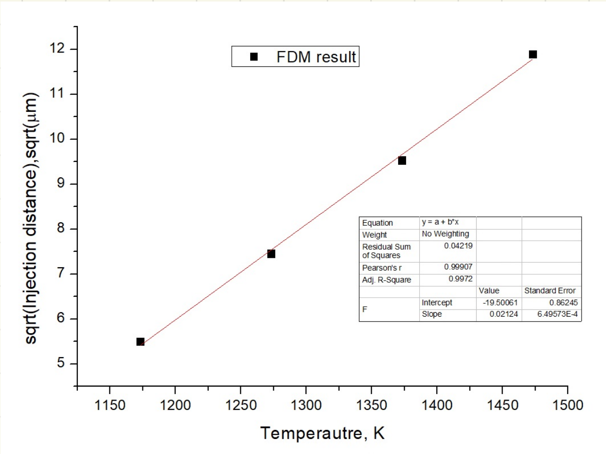
Injection time (hour)	Injection distance (μm)	$\sqrt{\text{Injection time}}$
0	0	0
2.4	30.3	1.55
4.8	42.5.	2.19
7.2	50.5	2.68
9.5	60.6	3.08



$l \propto \sqrt{t}$

(b)

Temp	D	Injection distance after 24hr (μm)
1173K	1.19547×10^{-9}	30.3
1273K	3.92924×10^{-9}	55.6.
1373K	1.08592×10^{-8}	90.9
1473K	2.61426×10^{-8}	141.4



$\sqrt{l} \propto T$

(C) $l \propto \sqrt{t}$, $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$ $\Rightarrow$ Injection distance는 $\sqrt{Dt}$에 비례

$\therefore\ l = a\sqrt{Dt}$ $\qquad \ln l = \ln a + \frac{1}{2}(\ln D + \ln t)$

$$\ln l_1 = \ln a + \frac{1}{2}(\ln D_1 + \ln t_1)$$
$$-\ \ln l_2 = \ln a + \frac{1}{2}(\ln D_2 + \ln t_2)$$

$$\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\left(\ln\left(\frac{D_1}{D_2}\right) + \ln\left(\frac{t_1}{t_2}\right)\right) \qquad D = D_0 \exp\left(-\frac{Q}{RT}\right)$$

For same time, $(t_1 = t_2)$ $\ln\left(\frac{l_1}{l_2}\right) = \frac{1}{2}\ln\left(D_0\exp\left(-\frac{Q}{RT_1}\right)\right) - \frac{1}{2}\ln\left(D_0\exp\left(-\frac{Q}{RT_2}\right)\right)$

$$= -\frac{Q}{2R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$T_1 = 1173K$, $T_2 = 1273K$ for $t = 2.4h$

$d_1 = 30.3\ \mu m$, $d_2 = 55.6\ \mu m$.

$$\ln\left(\frac{30.3}{55.6}\right) = -\frac{Q}{2.8314}\left(\frac{1}{1173} - \frac{1}{1273}\right)$$

$$\therefore Q = 150723\ J \simeq 147723\ J$$