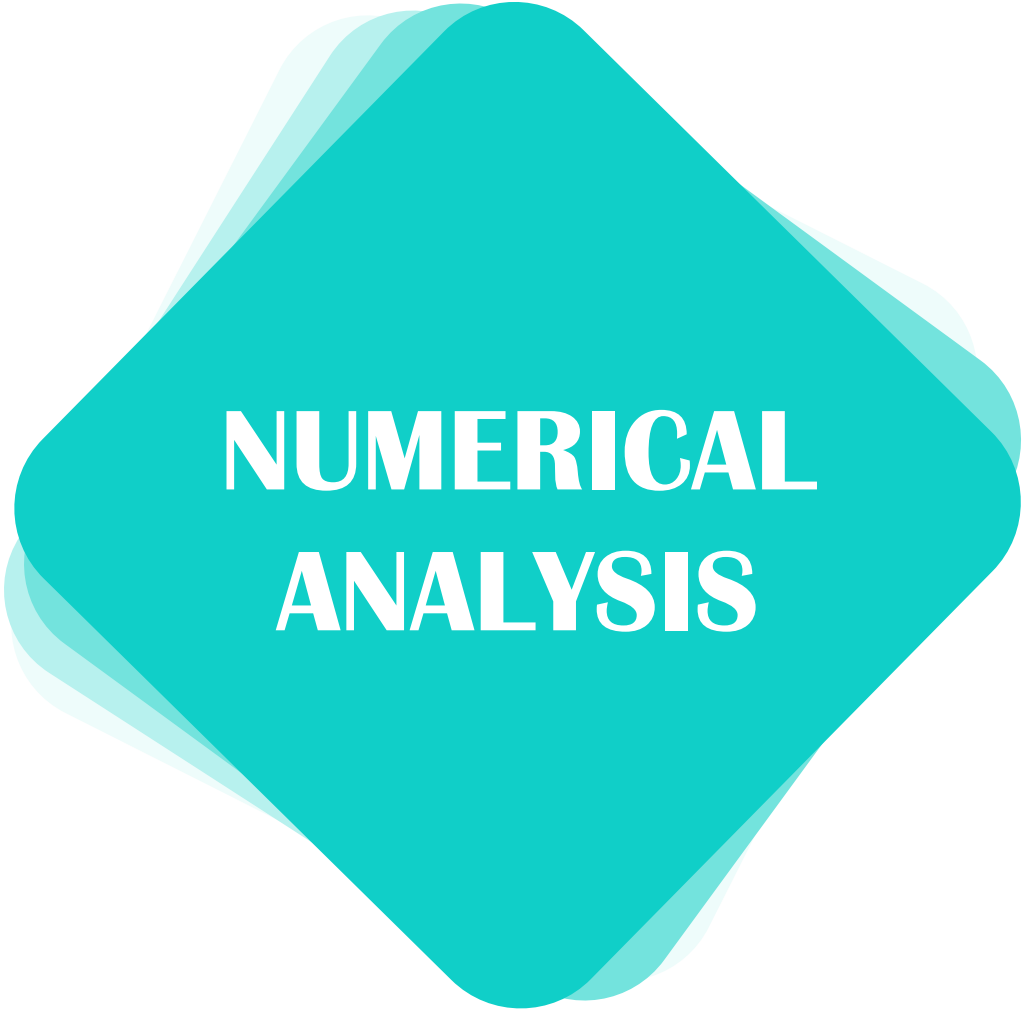




NUMERICAL ANALYSIS

6

조혜성

1. 주어진 데이터를 이용해 Phase Diagram 그리기

$$\mu_{Tk}^L = \Delta G_{Tk}^{Bcc \rightarrow L} + RT \ln x_{Tk}^L + (1 - x_{Tk}^L)^2 \{L[0] + L[1]T + (1 - 4x_{Tk}^L) * (L[2] + L[3]T)\}$$

$$\mu_{Tk}^{Fcc} = \Delta G_{Tk}^{Bcc \rightarrow Fcc} + RT \ln x_{Tk}^{Fcc} + (1 - x_{Tk}^{Fcc})^2 \{FCC[0] + FCC[1]T + (1 - 4x_{Tk}^{Fcc}) * (FCC[2] + FCC[3]T)\}$$

$$\mu_{Tk}^{Bcc} = RT \ln x_{Tk}^{Bcc} + (1 - x_{Tk}^{Bcc})^2 \{BCC[0] + BCC[1]T\}$$

$$\mu_{Ps}^L = \Delta G_{Ps}^{Fcc \rightarrow L} + RT \ln(1 - x_{Tk}^L) + (x_{Tk}^L)^2 \{L[0] + L[1]T + (3 - 4x_{Tk}^L) * (L[2] + L[3]T)\}$$

$$\mu_{Ps}^{Fcc} = RT \ln(1 - x_{Tk}^{Fcc}) + (x_{Tk}^{Fcc})^2 \{FCC[0] + FCC[1]T + (3 - 4x_{Tk}^{Fcc}) * (FCC[2] + FCC[3]T)\}$$

$$\mu_{Ps}^{Bcc} = \Delta G_{Ps}^{Fcc \rightarrow Bcc} + RT \ln(1 - x_{Tk}^{Bcc}) + (x_{Tk}^{Bcc})^2 \{BCC[0] + BCC[1]T\}$$

$$\begin{bmatrix} x_{Tk \text{ new}}^L \\ x_{Tk \text{ new}}^{FCC} \\ x_{Tk \text{ new}}^{BCC} \\ T_{\text{new}} \end{bmatrix} = \begin{bmatrix} x_{Tk}^L \\ x_{Tk}^{FCC} \\ x_{Tk}^{BCC} \\ T \end{bmatrix} - \begin{bmatrix} \frac{\partial f_{Tk}^{FCC \text{ to Liq}}}{\partial x_{Tk}^L} & \frac{\partial f_{Tk}^{FCC \text{ to Liq}}}{\partial x_{Tk}^{FCC}} & \frac{\partial f_{Tk}^{FCC \text{ to Liq}}}{\partial x_{Tk}^{BCC}} & \frac{\partial f_{Tk}^{FCC \text{ to Liq}}}{\partial T} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} f_{Tk}^{FCC \text{ to Liq}} \\ f_{Ps}^{FCC \text{ to Liq}} \\ f_{Tk}^{BCC \text{ to Liq}} \\ f_{Ps}^{BCC \text{ to Liq}} \end{bmatrix}$$

2. Algorithms

$$f_x AtoB = \mu_x^B - \mu_x^A$$

$$f_{Tk} FCCtoLiq, f_{Ps} FCCtoLiq, f_{Tk} BCCtoLiq, f_{Ps} BCCtoLiq, f_{Tk} BCCtoFCC, f_{Ps} BCCtoFCC$$

Matrix inversion

Newton's Method

범위 Invariant equilibrium $[T, T + 700], [T, T - 700]$
If Iteration > 998: break

Plot

3. Code

```
def fTkFCCtoLiq(L, FCC, T, xL, xfcc):  
    G = 6000 - 10 * T  
    fTk = G - GBF + R*T*(log(xL) - log(xfcc)) +(L[0]+L[1]*T +(1-4*x  
    return fTk  
def fPsFCCtoLiq(L, FCC, T, xL, xfcc):  
def fTkBCCtoLiq(L, BCC, T, x1, x2):  
def fPsBCCtoLiq(L, BCC, T, x1, x2):  
def fTkBCCtoFCC(BCC, FCC, T, x2, x3):  
def fPsBCCtoFCC(BCC, FCC, T, x2, x3):  
def makeJinv(L, B, T, x1, x2, x3, x4, k):  
def Newtons(T1, T2, a, s):
```

```
for i in range(0, 999, 1):  
    if s == 0:  
        Jinv = makeJinv(L, FCC, T,  
            x1_new = x1 - (Jinv[0][0]  
            x2_new = x2 - (Jinv[1][0]  
    elif s == 1:  
        Jinv = makeJinv(L, BCC, T,  
            x1_new = x1 - (Jinv[0][0]  
            x2_new = x2 - (Jinv[1][0]  
    elif s == 2:  
        Jinv = makeJinv(FCC, BCC,  
            x1_new = x1 - (Jinv[0][0]  
            x2_new = x2 - (Jinv[1][0]  
    elif s == 3:  
        Jinv = makeJinv(FCC, BCC,  
            x1_new = x1 - (Jinv[0][0]  
            x2_new = x2 - (Jinv[1][0]  
            x3_new = x3 - (Jinv[2][0]  
            x4_new = x4 - (Jinv[3][0]
```

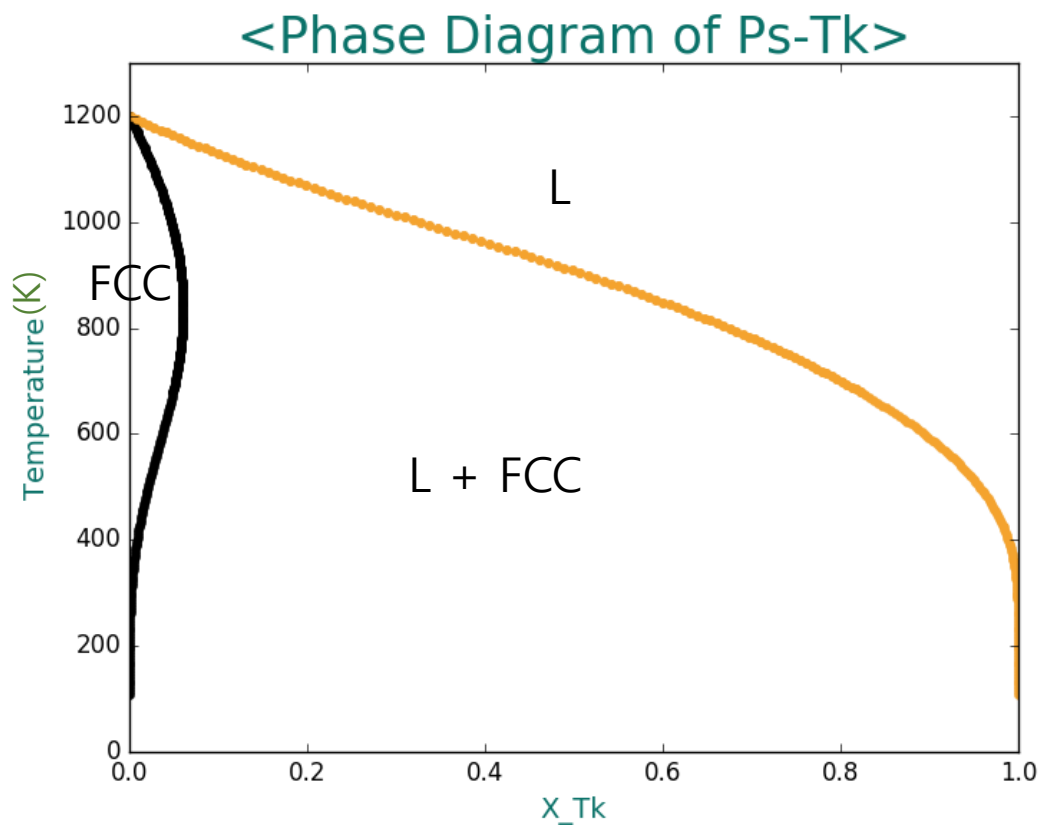
XTk_Liq: 0.8224 XTk_FCC: 0.0490

XTk_BCC: 0.6610

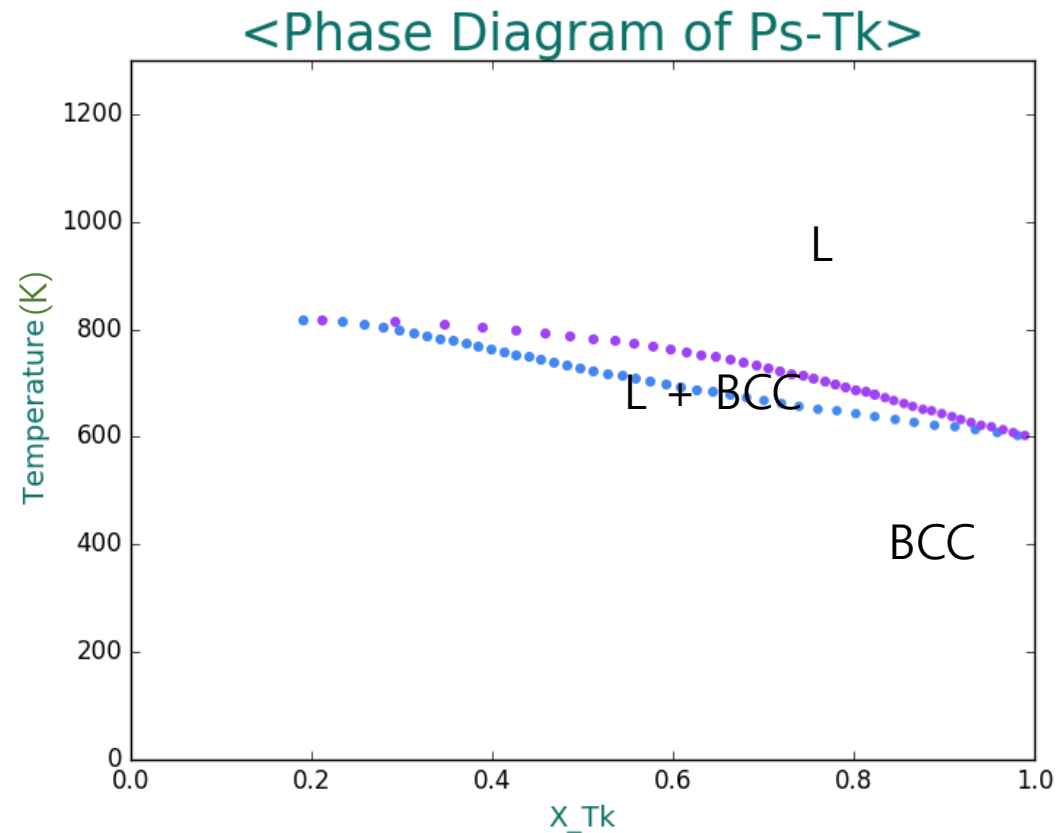
Temperature: 679.3177 i: 12

4. Result

$f_{Tk} FCCtoLiq, f_{Ps} FCCtoLiq$



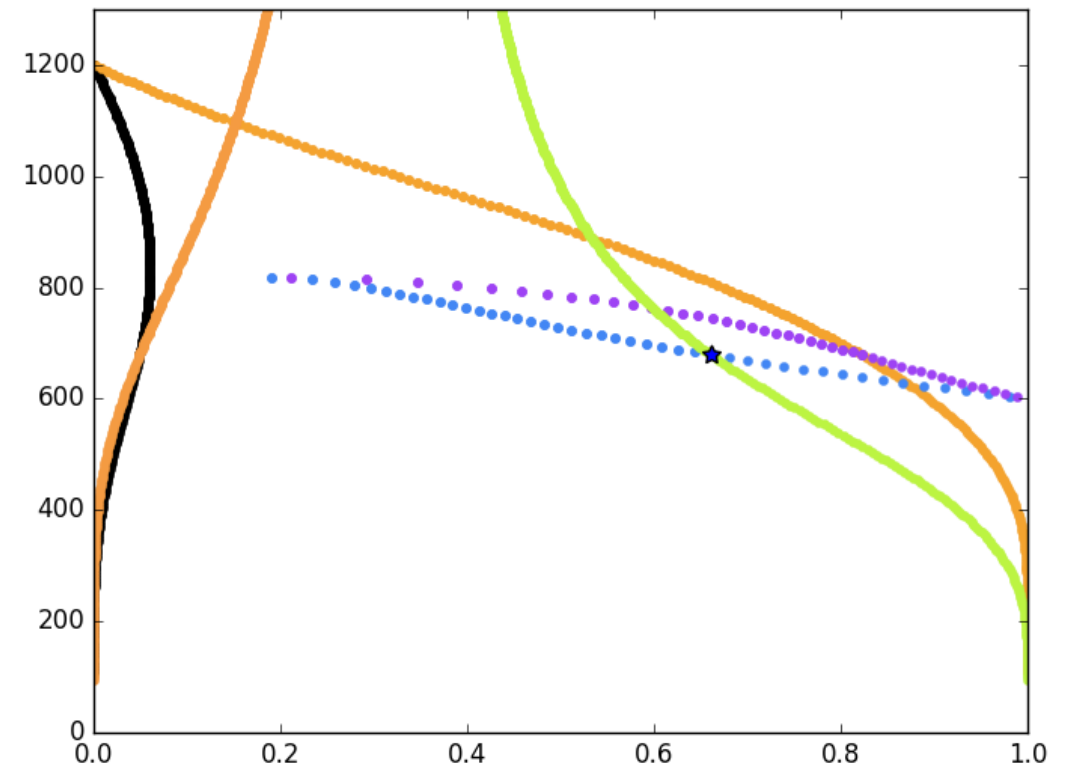
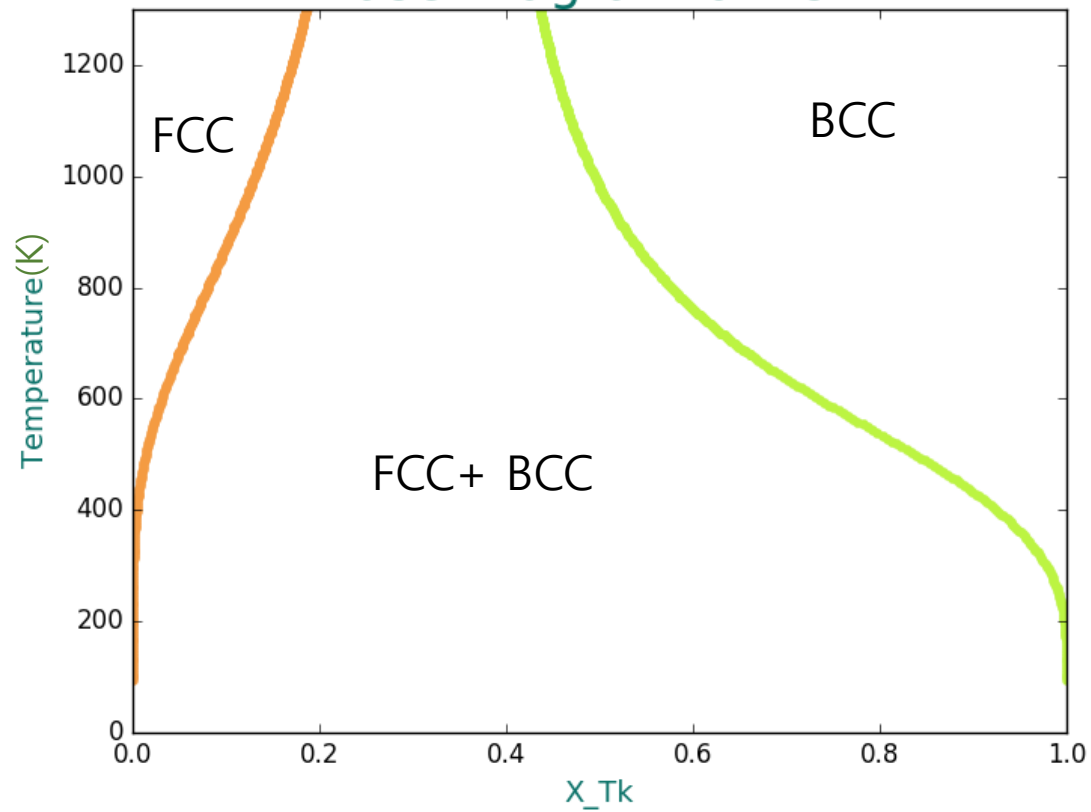
$f_{Tk} BCCtoLiq, f_{Ps} BCCtoLiq$



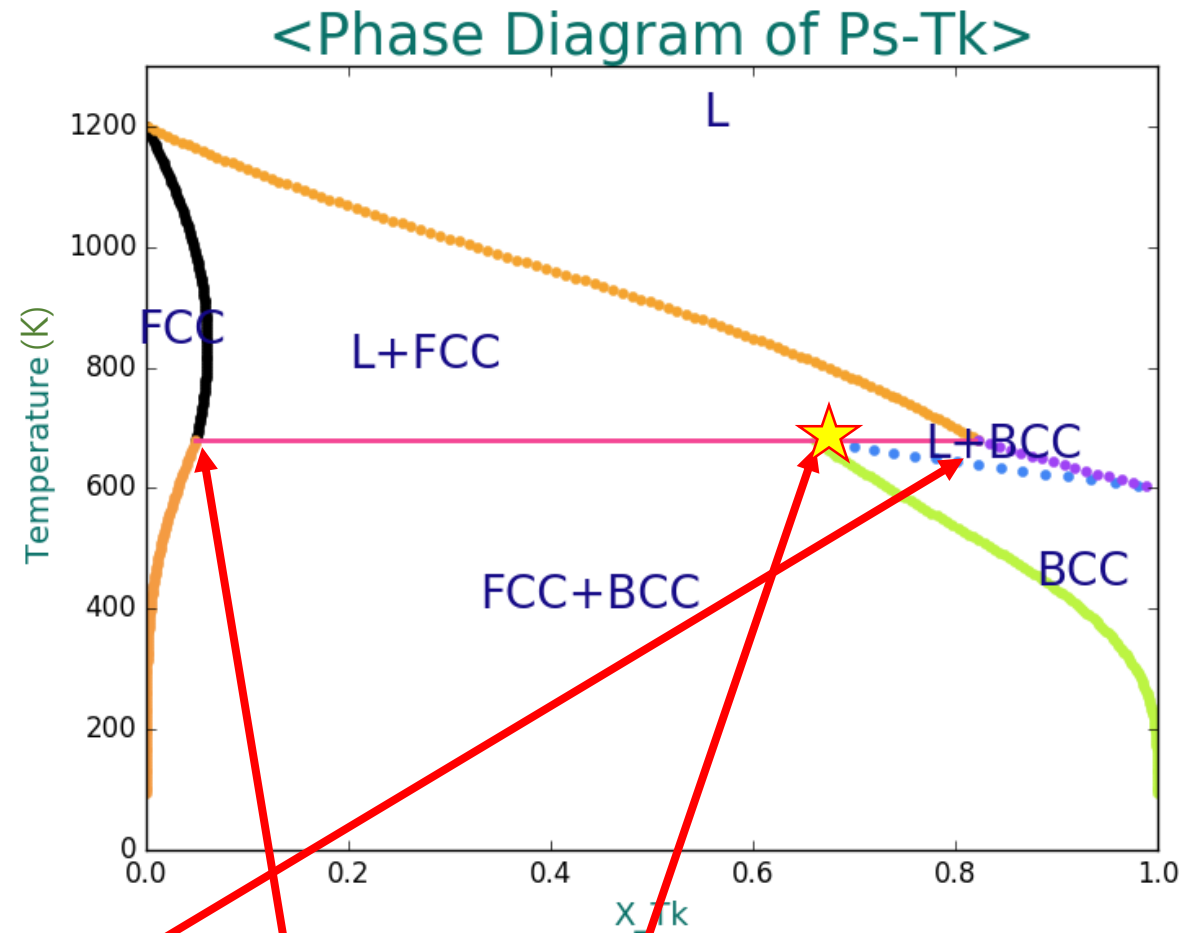
4. Result

$$f_{Tk} BCCtoFCC, f_{Ps} BCCtoFCC$$

<Phase Diagram of Ps-Tk>



4. Result

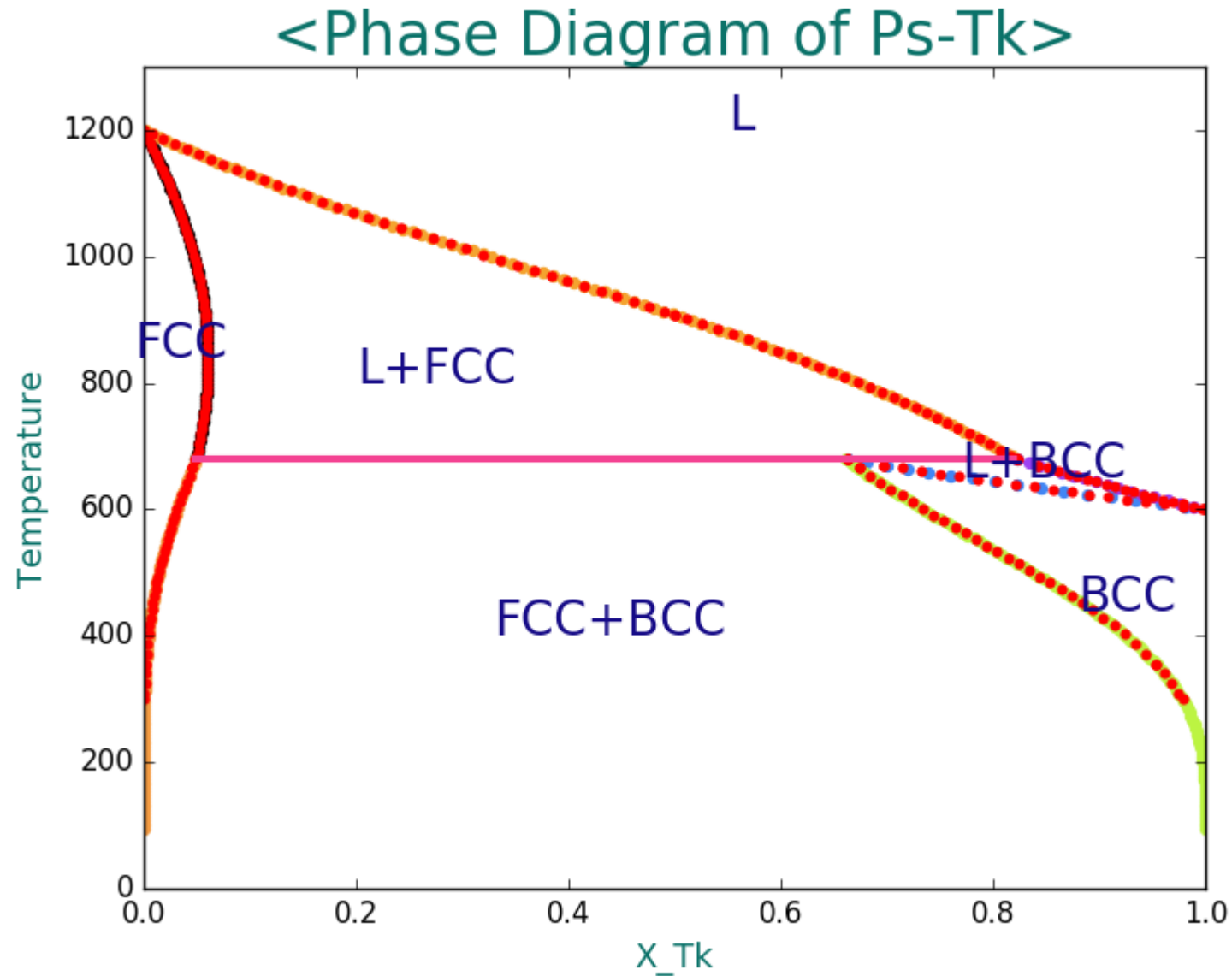


Peritectic point at
 $X_{Tk_BCC} = 0.66$
 $T = 679.3177 \text{ K}$

X_{Tk_Liq} : 0.8224 X_{Tk_FCC} : 0.0490 X_{Tk_BCC} : 0.6610 Temperature: 679.3177 i: 12

4. Result

Red dots =
주어진 data



5. 마지막 장입니다.

지금까지의 과제 중에 가장 프로그램을 돌리는데 오래 걸려서 기다리는 시간이 많았다

Binary Phase diagram에서는 각 성분의 녹는점을 기준으로 알고리즘을 돌렸는데,
이번 과제는 모든 상의 녹는점을 알 수 없었음

→ 모든 상이 존재하는 Peritectic Point를 기준, 위 아래로 데이터를 구함으로 해결

Plot하기 전에 Eutectic, Peritectic을 알 수는 없을까?

→ $X_{Tk_FCC} < X_{Tk_Liq} < X_{Tk_BCC}$ 이면 Eutectic

→ $X_{Tk_FCC} < X_{Tk_BCC} < X_{Tk_Liq}$ 이면 Peritectic으로 판단

이제 다음 과제 하러 가볼까요?ㅎㅎ