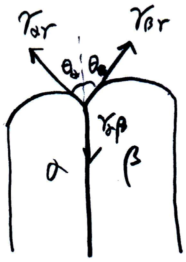
Phase transformation HW4

20152003
김용민

1. a)



만약 $\theta_\alpha = \theta_\beta = 90^\circ$ 이면 (no curvature),

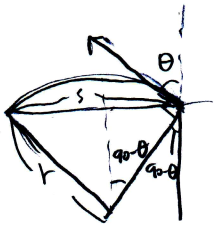
힘의 평형에 의한 식

$$\gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta = 0 = \gamma_{\alpha\beta}$$ 가 되는데,

이는 $\gamma_{\alpha\beta}$ 가 실제로 존재하는 것과 모순된다.

따라서 layer tip은 curvature를 갖는다.

b)



$$\sin(90-\theta) = \cos\theta = \frac{S}{2r}$$

$$\therefore\ r_\alpha = \frac{S_\alpha}{2\cos\theta_\alpha}, \quad r_\beta = \frac{S_\beta}{2\cos\theta_\beta}$$

그리고 force balance 에 의해

$$\gamma_{\alpha\gamma}\sin\theta_\alpha = \gamma_{\beta\gamma}\sin\theta_\beta, \quad \gamma_{\alpha\gamma}\cos\theta_\alpha + \gamma_{\beta\gamma}\cos\theta_\beta = \gamma_{\alpha\beta}$$

이식을 풀면 $$\cos\theta_\alpha = \frac{\gamma_{\alpha\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2}{2\gamma_{\alpha\gamma}\gamma_{\alpha\beta}} \quad \cos\theta_\beta = \frac{\gamma_{\beta\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2}{2\gamma_{\beta\gamma}\gamma_{\alpha\beta}}$$

$$\therefore\ r_\alpha = \frac{S_\alpha\gamma_{\alpha\gamma}\gamma_{\alpha\beta}}{\gamma_{\alpha\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2}, \quad r_\beta = \frac{S_\beta\gamma_{\beta\gamma}\gamma_{\alpha\beta}}{\gamma_{\beta\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2}$$

c) i) increase in Gibbs energy due to interfacial energy

$$\Delta G_{inter} = \frac{2}{S}\cdot\gamma_{\alpha\beta}\cdot V_m = \boxed{\frac{2\gamma_{\alpha\beta}V_m}{S_\alpha + S_\beta}}$$

ii) increase in Gibbs energy due to capillarity effect

$$\Delta G_{cap} = \frac{\gamma_{\alpha\gamma}}{r_\alpha}\cdot\left(\frac{S_\alpha}{S}V_m\right) + \frac{\gamma_{\beta\gamma}}{r_\beta}\cdot\left(\frac{S_\beta}{S}\cdot V_m\right)$$

$$= \left(\frac{\gamma_{\alpha\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\beta\gamma}^2}{S_\alpha\gamma_{\alpha\beta}}\cdot S_\alpha + \frac{\gamma_{\beta\gamma}^2 + \gamma_{\alpha\beta}^2 - \gamma_{\alpha\gamma}^2}{S_\beta\gamma_{\alpha\beta}}\cdot S_\beta\right)\cdot\frac{V_m}{S}$$

$$= \frac{2\gamma_{\alpha\beta}^2}{\gamma_{\alpha\beta}}\cdot\frac{V_m}{S} = \frac{2\gamma_{\alpha\beta}V_m}{S} = \boxed{\frac{2\gamma_{\alpha\beta}V_m}{S_\alpha + S_\beta}}$$