Numerical Method

Dept. Material Science & Engineering
Undergraduate Course (4th year)
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Homework #7

Numerical Integration

- Midpoint Rule

$$\int_a^b f(x)\,dx = (b-a)f\left(\frac{a+b}{2}\right) + \frac{f''(\xi)}{24}(b-a)^3.$$

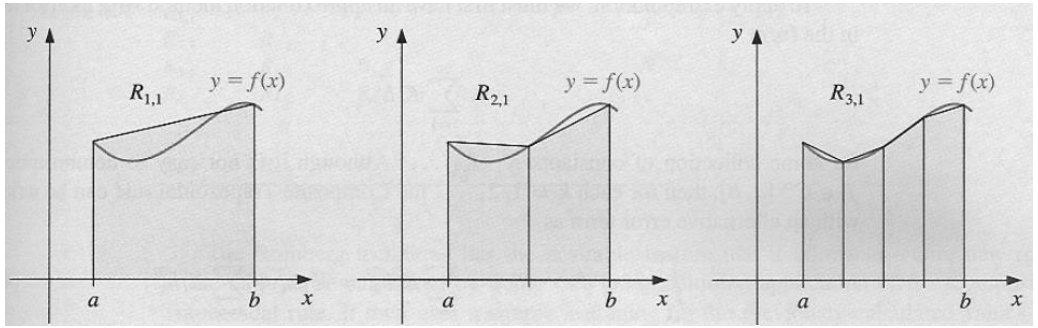
- Trapezoidal Rule

$$\int_a^b f(x)\,dx = (b-a)\frac{f(a)+f(b)}{2} - \frac{f''(\xi)}{12}(b-a)^3$$

- Simpson's Rule

$$\int_a^b f(x)\,dx = \frac{(b-a)}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] - \frac{f^{(4)}(\xi)}{2880}(b-a)^5$$

- Romberg Integration



Homework #7

- Midpoint Rule, Trapezoidal Rule, Simpson's Rule

```
double midpoint(int n, double x_ini, double x_last, int choose) { //n은 x_ini와 x_last 사이의 점 갯수
    double interval;
    int i;
    double sum=0;
    double x1, x2;
    interval = (x_last - x_ini) / (n + 1);
    x1 = x_ini;
    x2 = x_ini + interval;
    for (i = 0; i <= n; i++) {
        sum = sum + (x2 - x1)*func((x1 + x2) / 2);
        x1 = x2;
        x2 = x2 + interval;
    }
    if (choose == 0)
        return sum;
    else
        return fabs(100 * (sum - func_integ(x_ini, x_last)) / func_integ(x_ini, x_last));
}
```

n : 1 ~ 50
Initial x value : 0
Final x value : 0.8

1	1.900800	15.864759	1.068800	34.850455	1.623467	1.040312
2	1.770956	7.950039	1.369574	16.516560	1.637162	0.205494
3	1.716800	4.648895	1.484800	9.492848	1.639467	0.065020
4	1.690204	3.027724	1.539881	6.135345	1.640096	0.026632
5	1.675351	2.122365	1.570265	4.283260	1.640323	0.012843
6	1.666258	1.568051	1.588743	3.156899	1.640420	0.006933
7	1.660300	1.204893	1.600800	2.421977	1.640467	0.004064
8	1.656190	0.954373	1.609095	1.916356	1.640492	0.002537
9	1.653238	0.774408	1.615043	1.553810	1.640506	0.001665
10	1.651047	0.640842	1.619451	1.285096	1.640515	0.001137
11	1.649376	0.539020	1.622808	1.080448	1.640520	0.000803
12	1.648074	0.459637	1.625424	0.921023	1.640524	0.000583
13	1.647039	0.396562	1.627501	0.794424	1.640526	0.000433
14	1.646203	0.345620	1.629177	0.692226	1.640528	0.000329
15	1.645519	0.303890	1.630550	0.608542	1.640529	0.000254
16	1.644951	0.269280	1.631688	0.539157	1.640530	0.000199
17	1.644475	0.240258	1.632643	0.480992	1.640531	0.000159
18	1.644072	0.215684	1.633450	0.431752	1.640531	0.000128
19	1.643727	0.194694	1.634140	0.389701	1.640532	0.000104
20	1.643431	0.176624	1.634734	0.353505	1.640532	0.000086
21	1.643174	0.160957	1.635249	0.322127	1.640532	0.000071
22	1.642950	0.147284	1.635698	0.294747	1.640532	0.000059
23	1.642753	0.135282	1.636092	0.270714	1.640533	0.000050
24	1.642579	0.124688	1.636440	0.249504	1.640533	0.000043
25	1.642425	0.115292	1.636749	0.230693	1.640533	0.000036
26	1.642287	0.106918	1.637024	0.213931	1.640533	0.000031
27	1.642164	0.099425	1.637270	0.198931	1.640533	0.000027
28	1.642054	0.092692	1.637491	0.185455	1.640533	0.000024
29	1.641954	0.086621	1.637690	0.173303	1.640533	0.000021
30	1.641864	0.081127	1.637871	0.162308	1.640533	0.000018
31	1.641782	0.076139	1.638034	0.152326	1.640533	0.000016
32	1.641708	0.071598	1.638183	0.143238	1.640533	0.000014
33	1.641640	0.067451	1.638320	0.134939	1.640533	0.000012
34	1.641578	0.063654	1.638444	0.127341	1.640533	0.000011

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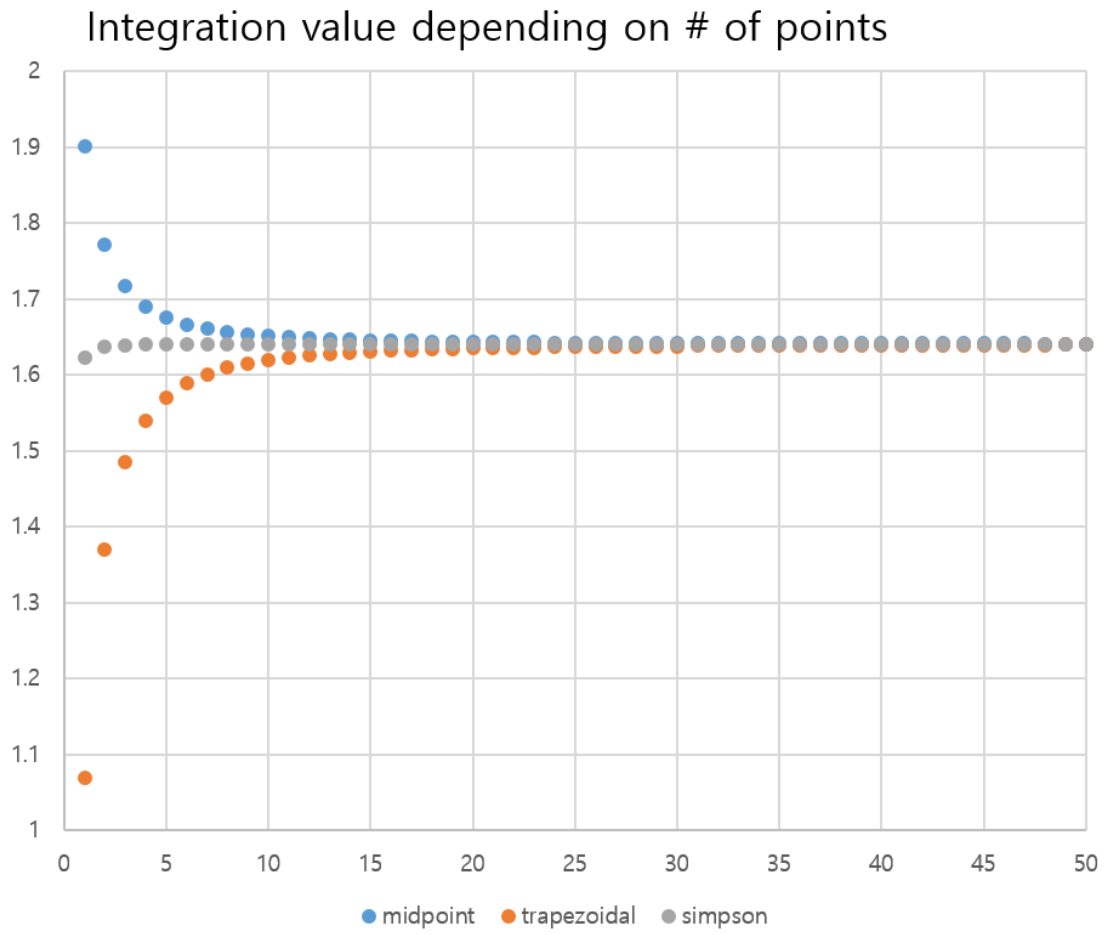
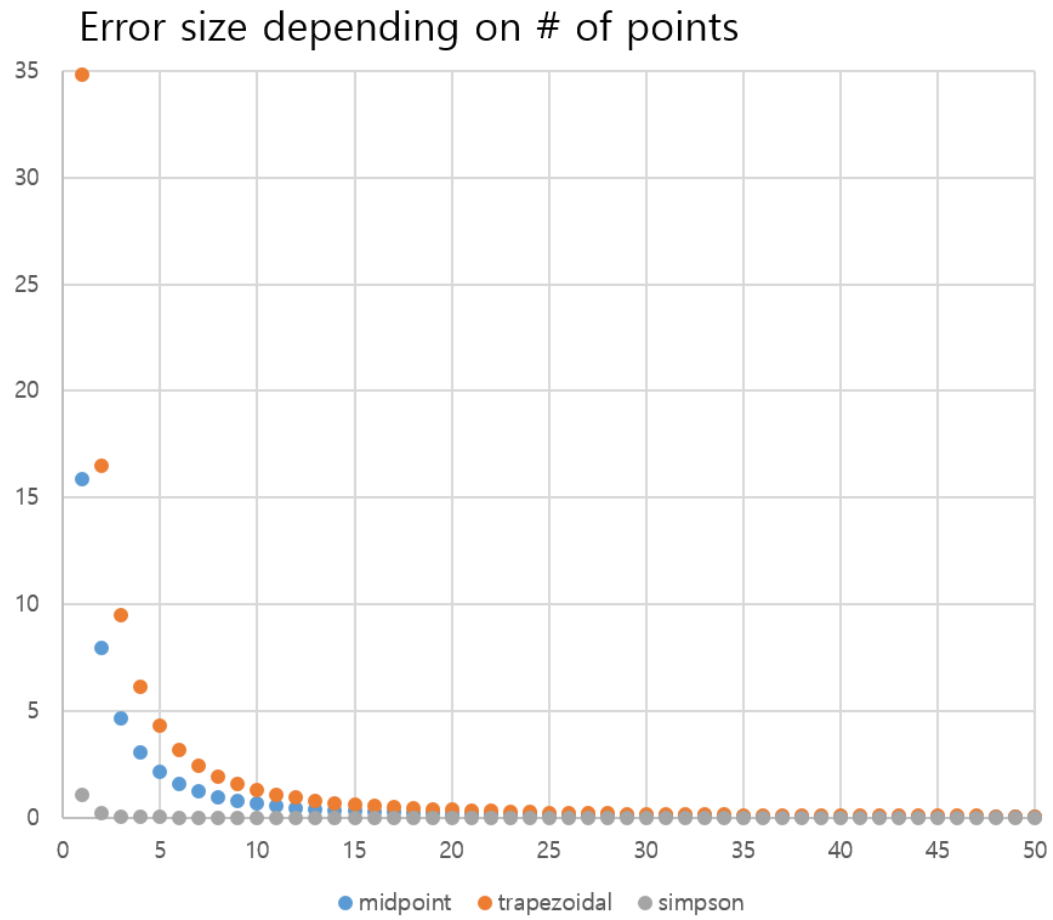
Real value : 1.64053334





Homework #7

- Romberg Integration

In general,

$$R_{k,1} = \frac{1}{2}\left\{ R_{k-1,1} + h_{k-1} \sum_{i=1}^{2^{k-2}} f(a + (2i-1)h_k \right\}$$

Extrapolation

$$R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1}$$

```
for (i = 1; i < n; i++) {
    H[i] = H[i - 1] / 2;
        sum = 0;
        for (k = 0; k < (pow(2, i - 1)); k++)
            sum = sum + func(x_ini + (2 * (k + 1) - 1)*H[i]);
        R[i][0] = (0.5) * (R[i - 1][0] + H[i - 1] * sum);
        for (j = 1; j <= i; j++)
            R[i][j] = R[i][j - 1] + (R[i][j - 1] - R[i - 1][j - 1]) / (pow(4, j) - 1);
}
```

Real value : 1.64053334

Integrated values

```
0.172800
1.068800        1.367467
1.484800        1.623467        1.640533

89.466840
34.850455       16.644993
9.492848        1.040312        0.000000
```

Error size

Initial x value : 0
Final x value : 0.8

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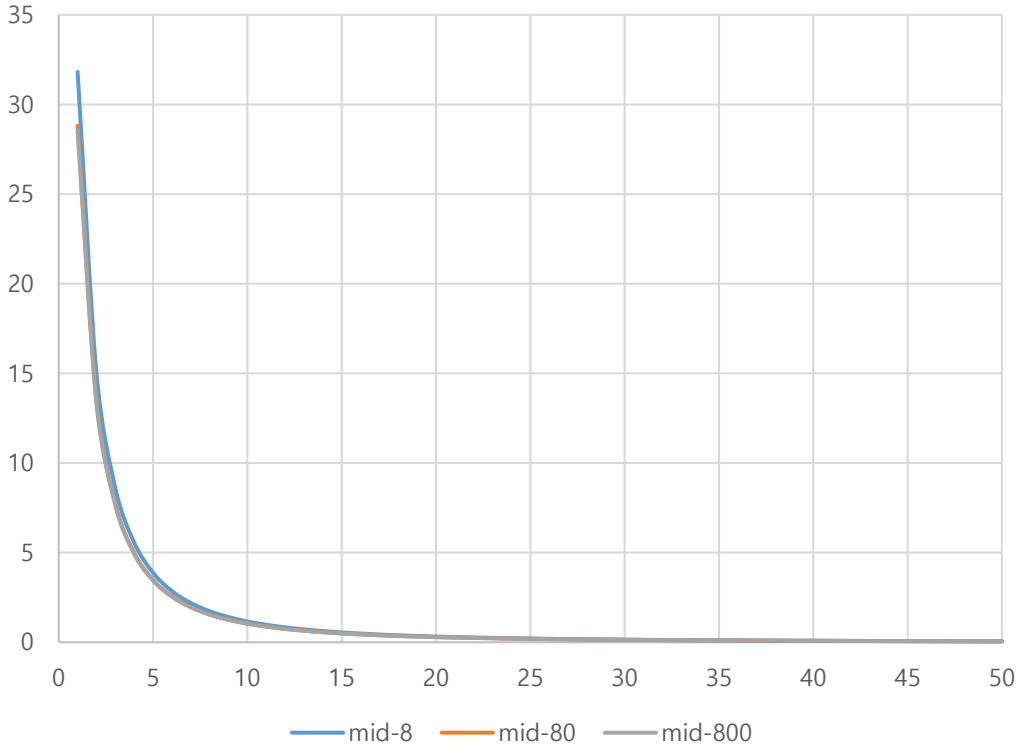
Range of Integration (x : 0~ 8, 80, 800)



Larger Range -> Smaller Error

Error Size of Midpoint (%)

	0~8	0~80	0~800
N =1	31.81372	28.80917	28.54466
2	14.99526	13.49467	13.36319
3	8.603321	7.726711	7.650017
4	5.556037	4.985371	4.935474
5	3.877187	3.477256	3.4423
6	2.856887	2.561449	2.535631

Error Size of Romberg (%)

218.858586
66.598371
17.392325

15.844966
0.990310

0.000000

0~8

201.705118
60.015679
15.603257

12.785866
0.799117

0.000000

0~80

200.168924
59.438359
15.446848

12.528171
0.783011

0.000000

0~800

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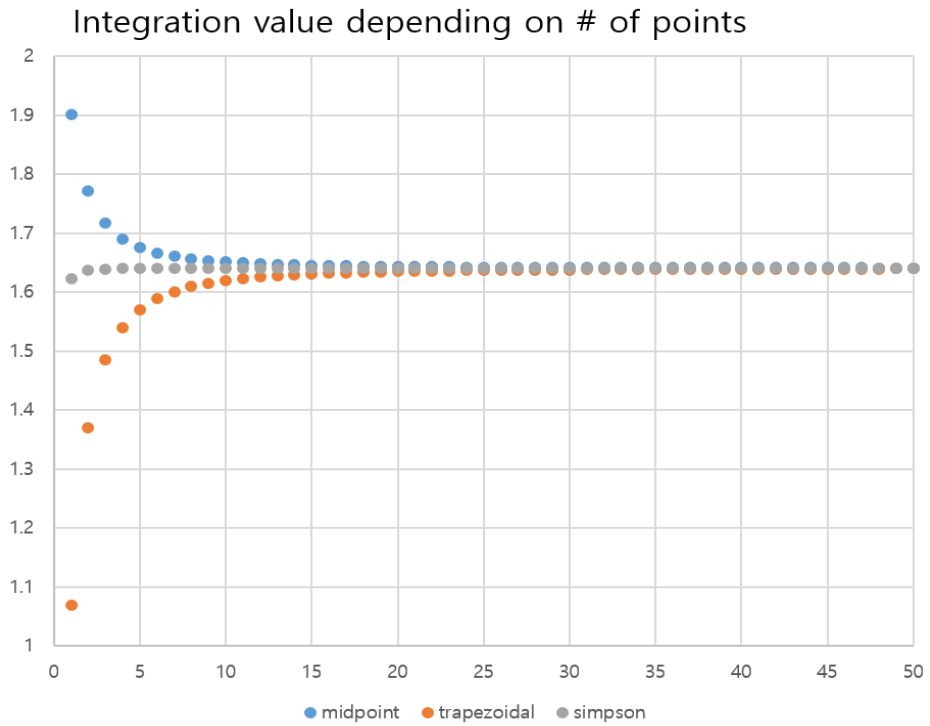
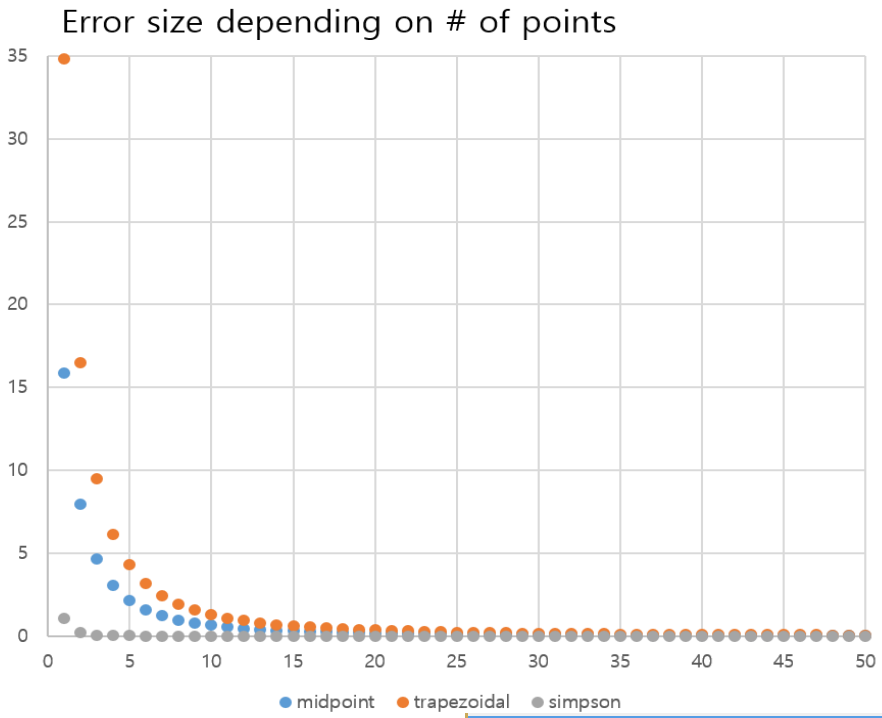
Question & Answer



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