Numerical Method

Dept. Material Science & Engineering
Undergraduate Course (4th year)
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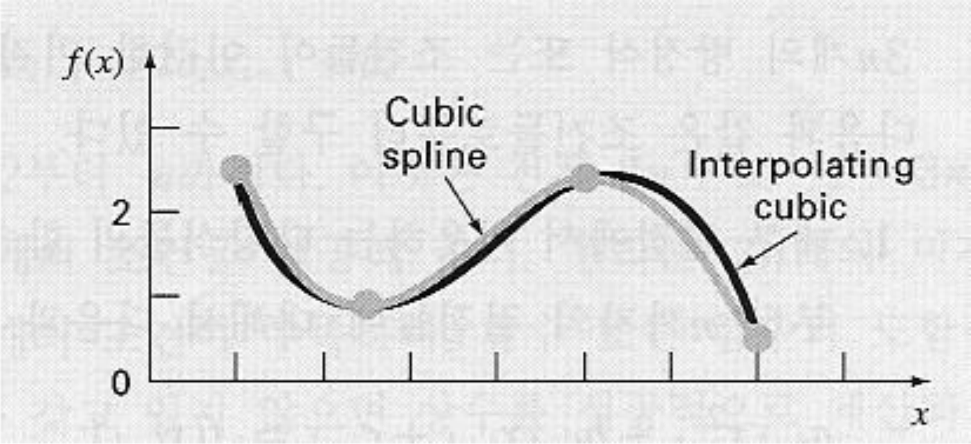
Homework #5

Interpolation -> Calculate ln2 with values of ln1, ln4 ……

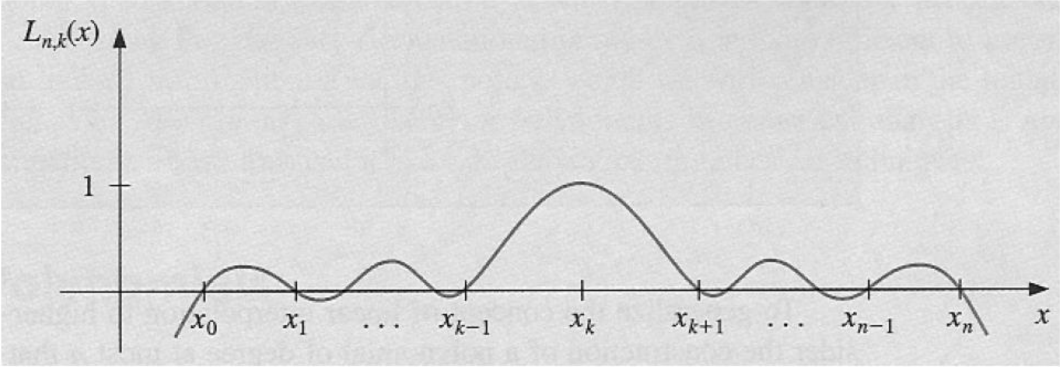
- Cubic Spline



$$f_i''(x) = f_i''(x_{i-1})\frac{x - x_i}{x_{i-1} - x_i} + f_i''(x_i)\frac{x - x_{i-1}}{x_i - x_{i-1}}$$

(with n-1 variables)

- Lagrange Polynomials



***n*th Lagrange Interpolating Polynomial**

$$P_n(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

where

$$L_{n,k}(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_{k-1})(x - x_{k+1})\cdots(x - x_n)}{(x_k - x_0)(x_k - x_1)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)}$$

for each $k = 0, 1, \ldots, n$.

Homework #5

- Cubic Spline

X (data)

$$\begin{bmatrix} x & lnx \\ \vdots & \vdots \\ \vdots & \vdots \end{bmatrix} \Rightarrow$$

A

$$\underbrace{(x_i - x_{i-1}) f''(x_{i-1}) + 2(x_{i+1} - x_{i-1}) f''(x_i) + (x_{i+1} - x_i) f''(x_{i+1})}_{A}$$
$$= \underbrace{\frac{6}{x_{i+1} - x_i}[f(x_{i+1}) - f(x_i)] + \frac{6}{x_i - x_{i-1}}[f(x_{i-1}) - f(x_i)]}_{B}$$

$$\overset{A}{\begin{bmatrix} 1 & & 0 \\ \vdots & & \vdots \\ 0 & & 1 \end{bmatrix}} \overset{Fi2}{\begin{bmatrix} f_1'' \\ \vdots \\ f_n'' \end{bmatrix}} = \overset{B}{\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}} \Rightarrow$$

Inverse Mat.

```
for (i = 1; i < n - 1; i++) {
    A[i][i-1] = X[i][0] - X[i-1][0];
    A[i][i] = 2 * (X[i+1][0] - X[i-1][0]);
    A[i][i+1] = X[i+1][0] - X[i][0];
    B[i][0] = deriv_value(X[i - 1][0], X[i][0], X[i + 1][0], X[i - 1][1], X[i][1], X[i + 1][1]);
}

double** inv_A = (double*)malloc(sizeof(double*)*(n));
for (i = 0; i < n; i++)
    *(inv_A + i) = (double*)malloc(sizeof(double)*(n));

for (i = 0; i < n; i++) {
    for (j = 0; j < n ; j++)
        *(*(inv_A + j) + i) = *(*(inverse_matrix(A, n, 0.000000000000000001) + j) + i);
}
```

$$f_i(x) = \underbrace{\frac{f_i''(x_{i-1})}{6(x_i - x_{i-1})}}_{Coeff\_1}(x_i - x)^3 + \frac{f_i''(x_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3$$
$$+ \left[\frac{f(x_{i-1})}{x_i - x_{i-1}} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x)$$
$$+ \left[\frac{f(x_i)}{x_i - x_{i-1}} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})$$

$\Rightarrow$ *Coefficient Matrix*

```
double** coeff = (double*)malloc(sizeof(double*)*(n-1)); //A
for (i = 0; i < n; i++)
    *(coeff + i) = (double*)malloc(sizeof(double)*(4));

for (i = 0; i < n - 1; i++) {
    coeff[i][0] = coeff_1(X[i][0], X[i + 1][0], Fi2[i][0]);
    coeff[i][1] = coeff_2(X[i][0], X[i + 1][0], Fi2[i+1][0]);
    coeff[i][2] = coeff_3(X[i][0], X[i + 1][0], X[i][1], Fi2[i][0]);
    coeff[i][3] = coeff_4(X[i][0], X[i + 1][0], X[i+1][1], Fi2[i+1][0]);
}
```

Homework #5

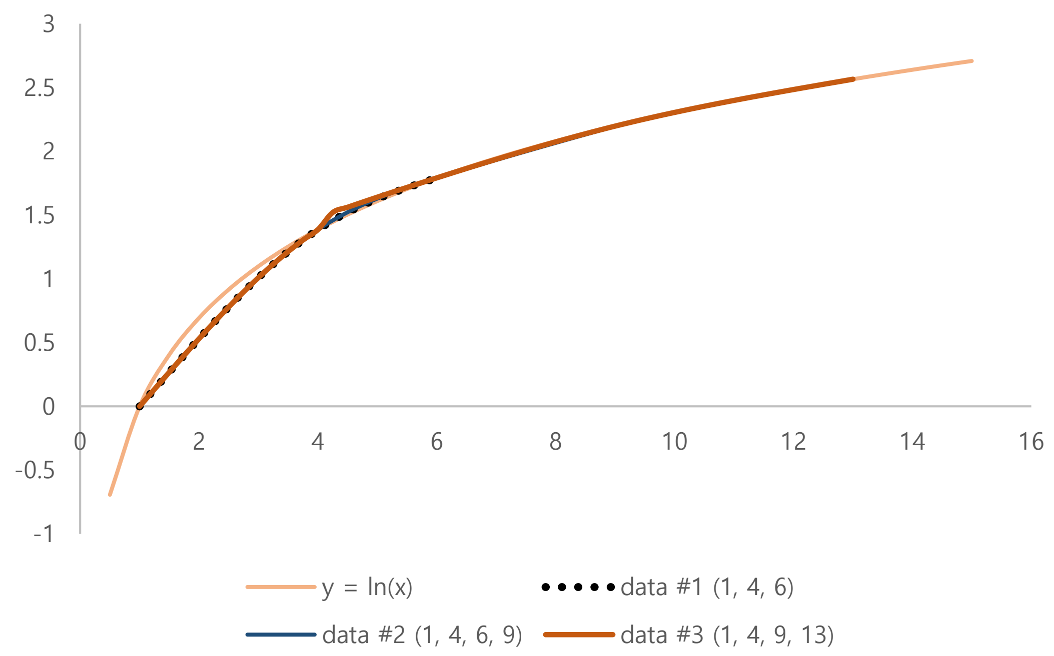
```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 3
Input data :
1 0
4 1.386294
6 1.791759
x =  1.000 ~  4.000 :
f1 = 0.00000( 4 - x)^3 + -0.00865(x -  1)^3 + 0.00000( 4 - x) + 0.53991(x -  1)
x =  4.000 ~  6.000 :
f2 = -0.01297( 6 - x)^3 + 0.00000(x -  4)^3 + 0.74502( 6 - x) + 0.89588(x -  4)
What do you find f(x) by interpolation? : 2
0.531262
계속하려면 아무 키나 누르십시오 . . .
```

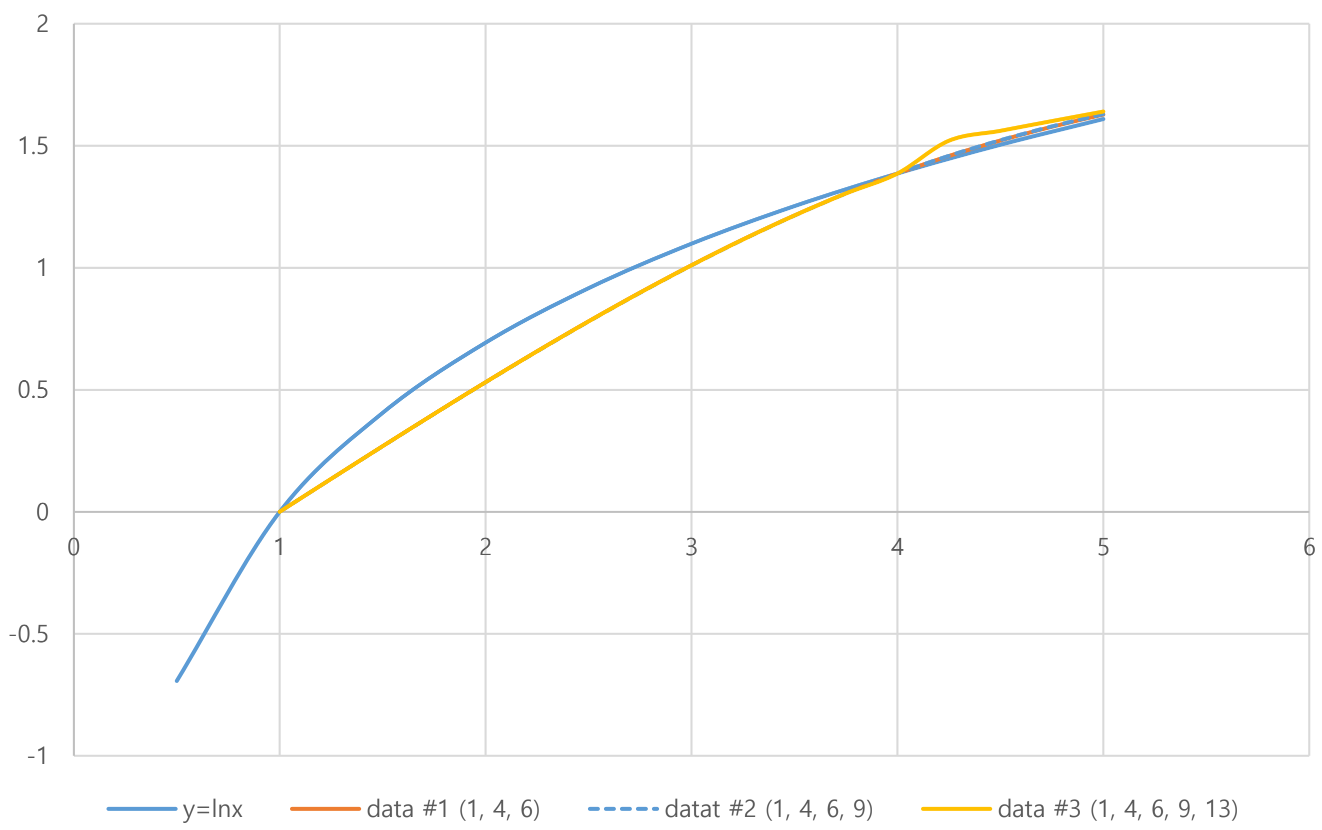
```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 4
Input data :
1 0
4 1.386294
6 1.791759
9 2.197225
x =  1.000 ~  4.000 :
f1 = 0.00000( 4 - x)^3 + -0.00854(x -  1)^3 + 0.00000( 4 - x) + 0.53893(x -  1)
x =  4.000 ~  6.000 :
f2 = -0.01280( 6 - x)^3 + -0.00082(x -  4)^3 + 0.74437( 6 - x) + 0.89915(x -  4)

x =  6.000 ~  9.000 :
f3 = -0.00055( 9 - x)^3 + 0.00000(x -  6)^3 + 0.60216( 9 - x) + 0.73241(x -  6)
What do you find f(x) by interpolation? : 2
0.530390
계속하려면 아무 키나 누르십시오 . . .
```

```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 5
Input data :
1 0
4 1.386294
6 1.791759
9 2.197225
13 2.564949
x =  1.000 ~  4.000 :
f1 = 0.00000( 4 - x)^3 + -0.00860(x -  1)^3 + 0.00000( 4 - x) + 0.53948(x -  1)
x =  4.000 ~  6.000 :
f2 = -0.01290( 6 - x)^3 + -0.00036(x -  4)^3 + 0.74473( 6 - x) + 0.89732(x -  4)

x =  6.000 ~  9.000 :
f3 = -0.00024( 9 - x)^3 + -0.00098(x -  6)^3 + 0.59941( 9 - x) + 0.74121(x -  6)

x =  9.000 ~ 13.000 :
f4 = -0.00073(13 - x)^3 + 0.00000(x -  9)^3 + 0.56104(13 - x) + 0.64124(x -  9)
What do you find f(x) by interpolation? : 2
0.530879
계속하려면 아무 키나 누르십시오 . . .
```

- Data : ln1, ln4 + ln6, ln9, ln13

Homework #5





$$ln2 = 0.693147$$

$$\Downarrow$$

$$ln2 = 0.531262\ (1,4,6) \qquad (23.36\%)$$
$$ln2 = 0.530390\ (1,4,6,9) \qquad (23.48\%)$$
$$ln2 = 0.530879\ (1,4,6,9,13) \qquad (23.41\%)$$

Homework #5

```
<Homework 5-1 : Interpolation by Cubic Splin
Input size : 5
Input data :
1 0
3 1.098612
4 1.386294
5 1.609438
6 1.791759
x =   1.000 ~   3.000 :
f1 = 0.00000( 3 - x)^3 + -0.02155(x -   1)^3
x =   3.000 ~   4.000 :
f2 = -0.04311( 4 - x)^3 + -0.00299(x -   3)^3

x =   4.000 ~   5.000 :
f3 = -0.00299( 5 - x)^3 + -0.00946(x -   4)^3

x =   5.000 ~   6.000 :
f4 = -0.00946( 6 - x)^3 + 0.00000(x -   5)^3
What do you find f(x) by interpolation? : 2
0.613964
계속하려면 아무 키나 누르십시오 . . .
```

```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 5
Input data :
1 0
3 1.0986294
5 1.609438
7 1.94591
9 2.197225
x =   1.000 ~   3.000 :
f1 = 0.00000( 3 - x)^3 + -0.01832(x -   1)^3 +
x =   3.000 ~   5.000 :
f2 = -0.01832( 5 - x)^3 + -0.00022(x -   3)^3

x =   5.000 ~   7.000 :
f3 = -0.00022( 7 - x)^3 + -0.00261(x -   5)^3

x =   7.000 ~   9.000 :
f4 = -0.00261( 9 - x)^3 + 0.00000(x -   7)^3 +
What do you find f(x) by interpolation? : 2
0.604260
계속하려면 아무 키나 누르십시오 . . .
```

```
<Homework 5-1 : Interpolation by Cubic Splin
Input size : 5
Input data :
1 0
3 1.098612
6 1.791759
9 2.197225
12 2.484907
x =   1.000 ~   3.000 :
f1 = 0.00000( 3 - x)^3 + -0.01605(x -   1)^3
x =   3.000 ~   6.000 :
f2 = -0.01070( 6 - x)^3 + 0.00030(x -   3)^3
x =   6.000 ~   9.000 :
f3 = 0.00030( 9 - x)^3 + -0.00117(x -   6)^3
x =   9.000 ~ 12.000 :
f4 = -0.00117(12 - x)^3 + 0.00000(x -   9)^3
What do you find f(x) by interpolation? : 2
0.597453
계속하려면 아무 키나 누르십시오 . . .
```

$$ln2 = 0.693147$$

$$\Downarrow$$

$$ln2 = 0.613964\ (1, 3, 4, 5, 6)$$
$$ln2 = 0.604260\ (1, 3, 5, 7, 9)$$
$$ln2 = 0.597453\ (1, 3, 6, 9, 12)$$

```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 5
Input data :
1 0
4 1.386294
50 3.912023
100 4.60517
200 5.398317
x =  1.000 ~  4.000 :
f1 = 0.00000( 4 - x)^3 + -0.00153(x -  1)^3 + 0.
x =  4.000 ~ 50.000 :
f2 = -0.00010(50 - x)^3 + 0.00002(x -  4)^3 + 0.
x = 50.000 ~ 100.000 :
f3 = 0.00002(100 - x)^3 + -0.00000(x - 50)^3 + 0
)
x = 100.000 ~ 200.000 :
f4 = -0.00000(200 - x)^3 + 0.00000(x - 100)^3 +
00)
What do you find f(x) by interpolation? : 2
0.474345
계속하려면 아무 키나 누르십시오 . . .
```

```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 5
Input data :
1 0
4 1.386294
1000 6.907755
10000 9.21034
100000 11.51293
x =   1.000 ~   4.000 :
f1 = 0.00000( 4 - x)^3 + -0.00008(x -   1)^3 + 0
x =   4.000 ~ 1000.000 :
f2 = -0.00000(1000 - x)^3 + 0.00000(x -   4)^3 +
 4)
x = 1000.000 ~ 10000.000 :
f3 = 0.00000(10000 - x)^3 + -0.00000(x - 1000)^
(x - 1000)
x = 10000.000 ~ 100000.000 :
f4 = -0.00000(100000 - x)^3 + 0.00000(x - 10000
13(x - 10000)
What do you find f(x) by interpolation? : 2
0.462723
계속하려면 아무 키나 누르십시오 . . .
```

$\cong$

```
<Homework 5-1 : Interpolation by Cubic Spline
Input size : 2
Input data :
1 0
4 1.386294
x =   1.000 ~   4.000 :
f1 = 0.00000( 4 - x)^3 + 0.00000(x -   1)^3 + 0.
What do you find f(x) by interpolation? : 2
0.462098
계속하려면 아무 키나 누르십시오 . . .
```

Homework #5

- Lagrange Polynomials

```c
double** L = (double*)malloc(sizeof(double*)*(n));
for (i = 0; i < n; i++)
    *(L + i) = (double*)malloc(sizeof(double)*(1));

double multi;
for (i = 0; i < n; i++) {
    multi = 1;
    for (j = 0; j < n; j++) {
        if (i != j)
            multi = multi * ((x - X[j][0]) / (X[i][0] - X[j][0]));
    }
    L[i][0] = multi;
}
```

***n*th Lagrange Interpolating Polynomial**

$$P_n(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x),$$

where

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

for each $k = 0, 1, \ldots, n$.

```
<Homework 5-2 : Interpolation by Lagrange Polynomials
Input size : 3
Input data :
1 0
4 1.386294
6 1.791759

Which x do you want to find by interpolation? : 2
f3 = 0.533333 f(x0) + 0.666667 f(x1) + -0.200000 f(x2)
0.565844계속하려면 아무 키나 누르십시오 . . .
```

$ln2 = 0.565844\ (1, 4, 6)$
(18.37%)

```
<Homework 5-2 : Interpolation by Lagrange Polynomials
Input size : 4
Input data :
1 0
4 1.386294
6 1.791759
9 2.197225

Which x do you want to find by interpolation? : 2
f4 = 0.466667 f(x0) + 0.933333 f(x1) + -0.466667 f(x2)
0.604202계속하려면 아무 키나 누르십시오 . . .
```

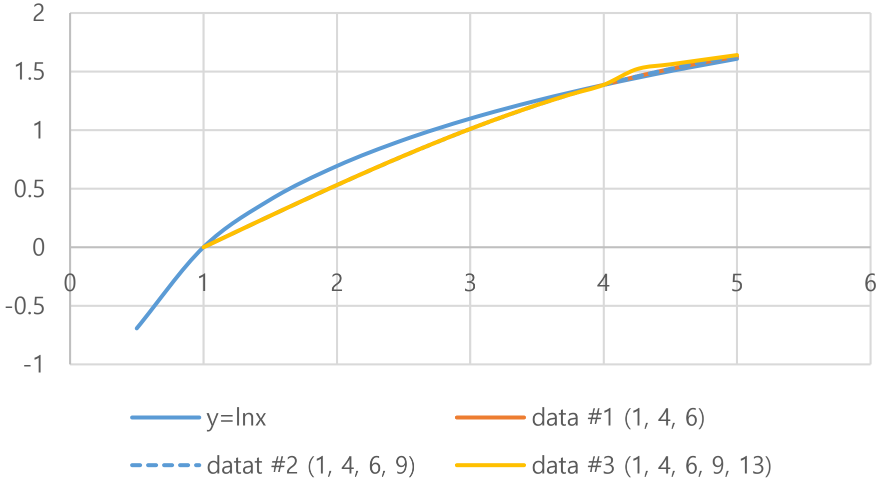
$ln2 = 0.604202\ (1, 4, 6, 9)$
(12.83%)

```
<Homework 5-2 : Interpolation by Lagrange Polynomials
Input size : 5
Input data :
1 0
4 1.386294
6 1.791759
9 2.197225
13 2.564949

Which x do you want to find by interpolation? : 2
f5 = 0.427778 f(x0) + 1.140741 f(x1) + -0.733333 f(x2)
519 f(x4)
0.622771계속하려면 아무 키나 누르십시오 . . .
```

$ln2 = 0.622771\ (1, 4, 6, 9\ 13)$
(10.15%)

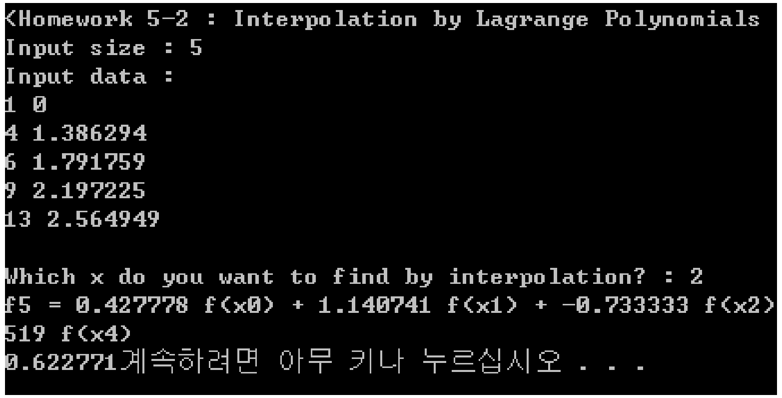
Question & Answer

- Cubic Spline



- $ln1$, $ln4$를 제외한 다른 값을 대입했을 때, $ln6$, $ln9$, $ln13$ 등의 데이터가 많아져도 $ln2$의 값이 약 0.53 근처에서 나타난다. (데이터 수에 따른 경향성이 안보임)
- $ln1$, $ln4$ 사이의 값인 $ln3$을 대입했을 때 더 높은 정확성을 보인다.
- x의 간격이 좁을수록 더 높은 정확성을 보인다.
- $ln1$, $ln4$ 이외에 $ln1000$, $ln10000$과 같은 데이터를 넣을 시 spline이 직선형으로 나타난다.

- Lagrange Polynomials



```
<Homework 5-2 : Interpolation by Lagrange Polynomials
Input size : 5
Input data :
1 0
4 1.386294
6 1.791759
9 2.197225
13 2.564949

Which x do you want to find by interpolation? : 2
f5 = 0.427778 f(x0) + 1.140741 f(x1) + -0.733333 f(x2)
519 f(x4)
0.622771계속하려면 아무 키나 누르십시오 . . .
```

- $ln1$, $ln4$ 이외의 데이터가 많아질 수록 더 높은 정확성을 보인다.
- $ln2$를 구하기 위해 똑같은 데이터를 대입했을 시, Lagrange Polynomials이 더 높은 정확성을 띈다.