Numerical Analysis For Materials

Homework #8

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Homework #8

IVP of simultaneous 1st ODE

1. Theory

4th RK

$$\frac{dy}{dx} = F(x,y,z) \qquad \frac{dz}{dx} = G(x,y,z)$$

$$K_1 = F(x_n, y_n, z_n)\Delta x$$
$$L_1 = G(x_n, y_n, z_n)\Delta x$$
$$K_2 = F(x_n + \Delta x/2, y_n + K_1/2, z_n + L_1/2)\Delta x$$
$$L_2 = G(x_n + \Delta x/2, y_n + K_1/2, z_n + L_1/2)\Delta x$$
$$K_3 = F(x_n + \Delta x/2, y_n + K_2/2, z_n + L_2/2)\Delta x$$
$$L_3 = G(x_n + \Delta x/2, y_n + K_2/2, z_n + L_2/2)\Delta x$$
$$K_4 = F(x_n + \Delta x, y_n + K_3, z_n + L_3)\Delta x$$
$$L_4 = G(x_n + \Delta x, y_n + K_3, z_n + L_3)\Delta x$$

$$y_{n+1} = y_n + \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$$
$$z_{n+1} = z_n + \frac{L_1 + 2L_2 + 2L_3 + L_4}{6}$$

(It is general case, and it depends on variables)

1. Theory

Heun

- Heun Method
  구간의 시작점과 끝점에서 도함수를 구하고 두 값의 평균을 취함

  예측자 방정식 (predictor equation)

  $$y_{i+1}^{o} = y_i + f(x_i, y_i)h$$

  y_{i+1} 의 추정값을 사용 구간의 끝점에서의 도함수를 계산

  $$y'_{i+1} = f(x_{i+1}, y_{i+1}^{o})$$

  구간 평균 기울기를 계산

  $$\bar{y}' = \frac{y'_i + y'_{i+1}}{2} = \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{o})}{2}$$

  평균 기울기를 이용하여 y_{i+1} 값을 근사

  $$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{o})}{2}h$$

  (수정자 방정식 corrector equation)

2. Programmed code

* Description of Program

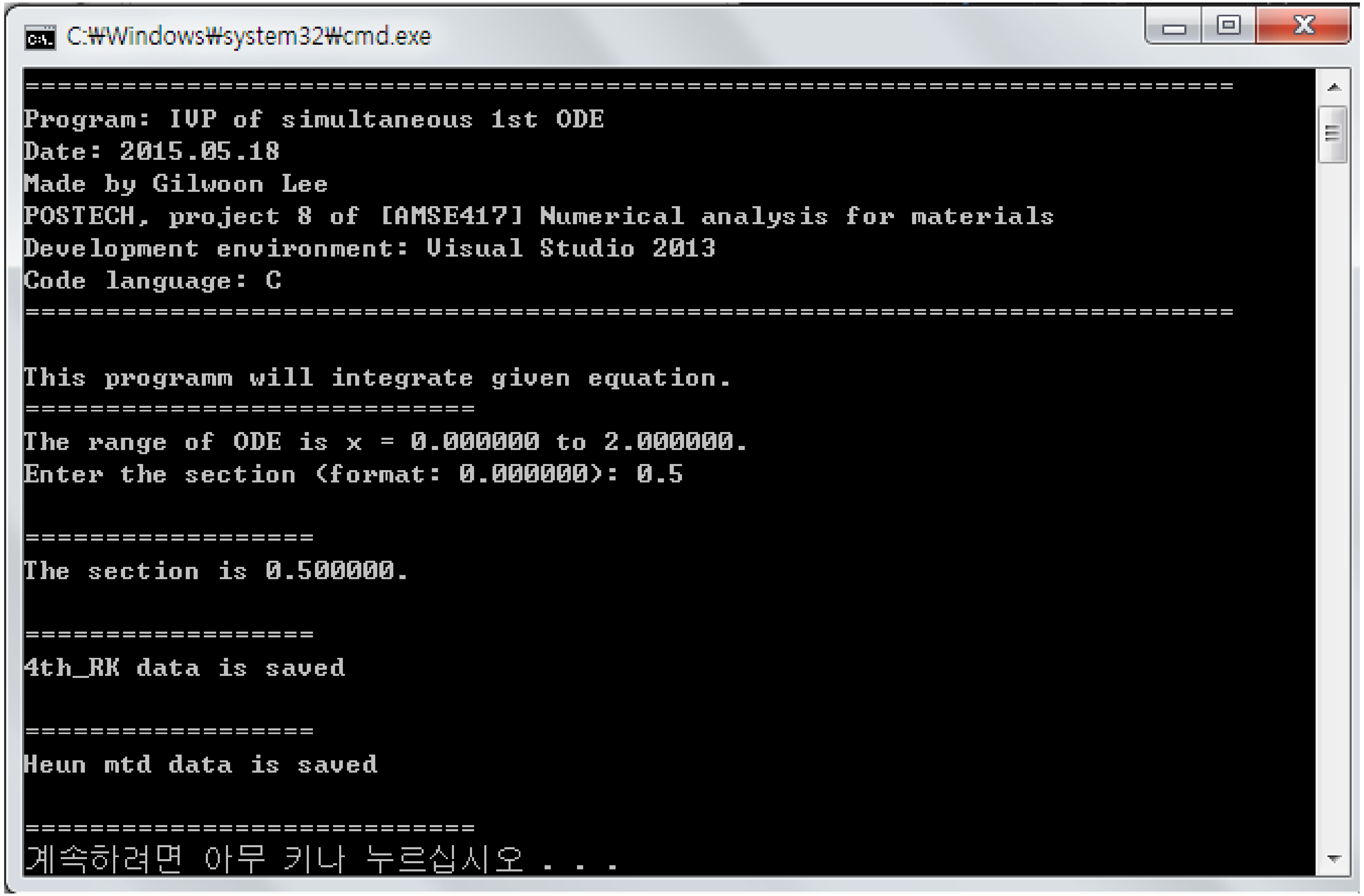
3. Part of functions

```
void _4th_RK(FILE *file, double n)
{
double x = MIN;
double y = 4.0;
double z = 6.0;
double k_1, k_2, k_3, k_4, l_1, l_2, l_3, l_4;

fprintf(file, "%.6f %.6f %.6f\n", x, y, z);
for (x = MIN + n; x <= MAX; x += n)
{
k_1 = F(y)*n;
k_2 = F(y + k_1 / 2)*n;
k_3 = F(y + k_2 / 2)*n;
k_4 = F(y + k_3)*n;
y += (k_1 + 2 * k_2 + 2 * k_3 + k_4) / 6;

l_1 = G(y, z)*n;
l_2 = G(y + l_1 / 2, z + l_1 / 2)*n;
l_3 = G(y + l_2 / 2, z + l_2 / 2)*n;
l_4 = G(y + l_3, z + l_3)*n;
z += (l_1 + 2 * l_2 + 2 * l_3 + l_4) / 6;

fprintf(file, "%.6f %.6f %.6f\n", x, y, z);
}
}
```

```
void Heun(FILE *file, double n)
{
double x = MIN;
double y = 4.0;
double z = 6.0;
double y_temp, z_temp;
double temp_1 = 100000;
double temp_2 = 100000;
int good = 1;

fprintf(file, "%.6f %.6f %.6f\n", x, y, z);
for (x = MIN + n; x <= MAX; x += n)
{
y_temp = y;
z_temp = z;
y_temp += F(y)*n;
z_temp += G(y, z)*n;
y += (F(y) + F(y_temp))*n / 2;
z += (G(y, z) + G(y_temp, z_temp))*n / 2;
fprintf(file, "%.6f %.6f %.6f\n", x, y, z);
}
}
```

3. Result & Conclusion

1. 실행 결과



C:₩Windows₩system32₩cmd.exe

```
===========================================================================
Program: IVP of simultaneous 1st ODE
Date: 2015.05.18
Made by Gilwoon Lee
POSTECH, project 8 of [AMSE417] Numerical analysis for materials
Development environment: Visual Studio 2013
Code language: C
===========================================================================

This programm will integrate given equation.
============================
The range of ODE is x = 0.000000 to 2.000000.
Enter the section (format: 0.000000): 0.5

==================
The section is 0.500000.

==================
4th_RK data is saved

==================
Heun mtd data is saved

============================
계속하려면 아무 키나 누르십시오 . . .
```

3. Result & Conclusion

1. 실행 결과

To get actual data, used wolfram alpha for getting original function

Input interpretation

$$\left\{\frac{\partial y(x)}{\partial x} = -0.5\, y(x),\ \frac{\partial z(x)}{\partial x} = 4 - 0.3\, z(x) - 0.1\, y(x),\ y(0) = 4,\ z(0) = 6\right\}$$

Differential equation solutions

$$y(x) = 4.\, e^{-0.5\, x}$$

$$z(x) = 13.3333 + 2.\, e^{-0.5\, x} - 9.33333\, e^{-0.3\, x}$$

$$y = 4e^{-0.5x}$$

$$z = \frac{40}{3} + 2e^{-0.5x} - \frac{28}{3}e^{-0.3x}$$

Real data		
x	y	z
0	4	6
0.5	3.115203	6.85766
1	2.426123	7.632091
1.5	1.889466	8.32687
2	1.471518	8.94685

3. Result & Conclusion

1. 실행 결과

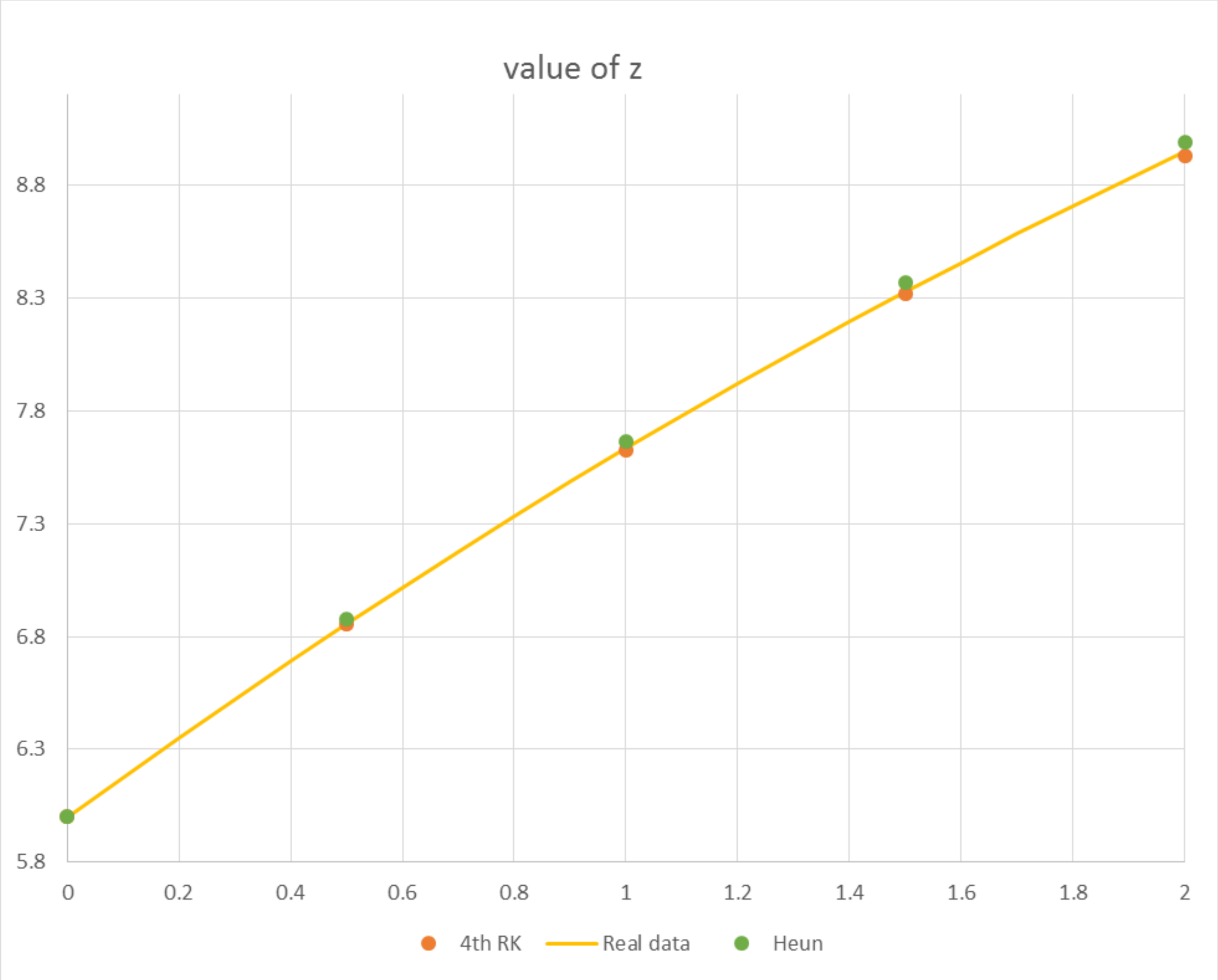
More Accurate

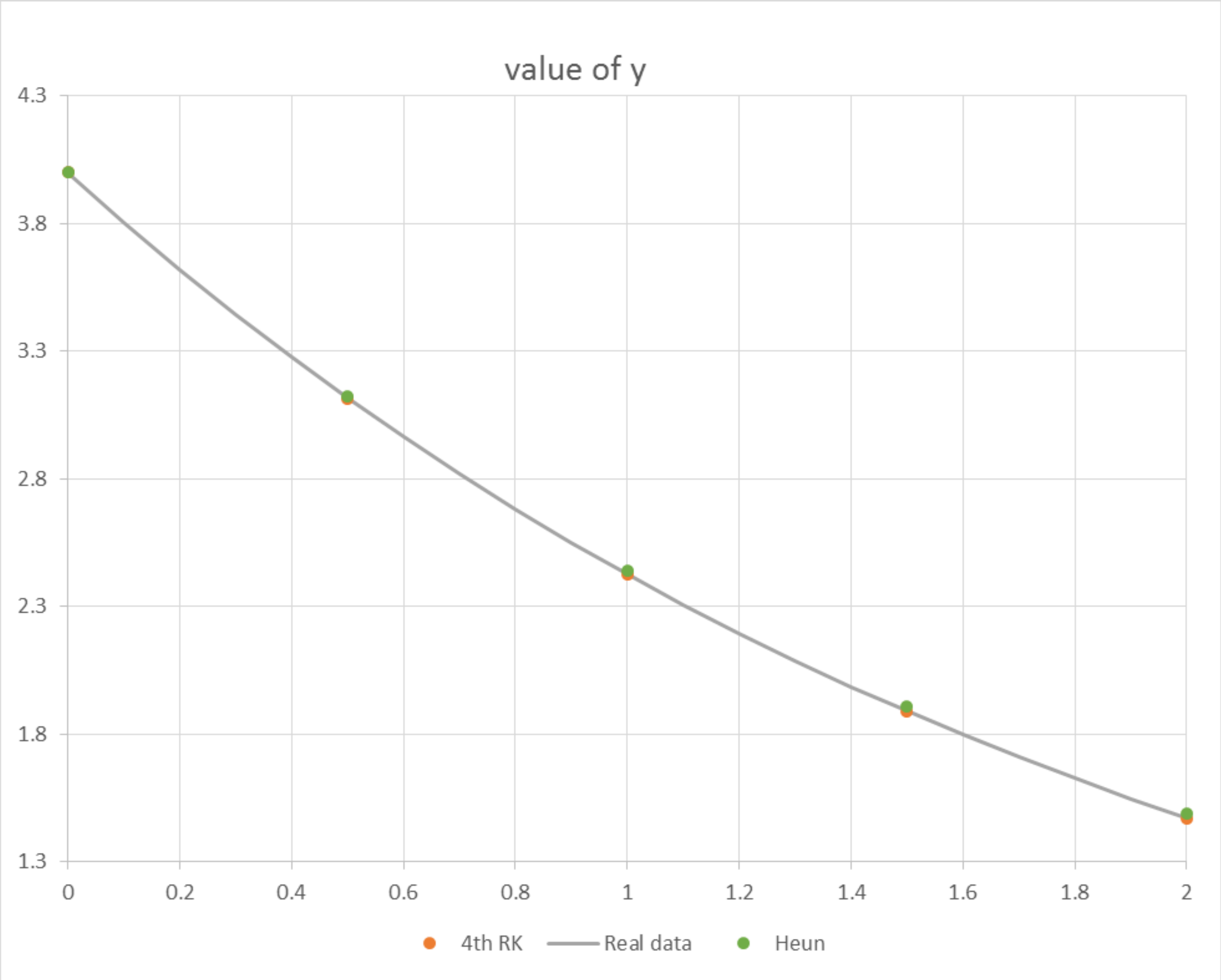
4th RK				
x	y	z	Error x(%)	Error y(%)
0	4	6	0	0
0.5	3.115234	6.855795	0.000991	-0.0272
1	2.426171	7.62647	0.001993	-0.07365
1.5	1.889523	8.316691	0.003006	-0.12225
2	1.471577	8.932017	0.004025	-0.16579

Heun				
x	y	z		
0	4	6	0	0
0.5	3.125	6.879375	0.314486	0.316647
1	2.441406	7.666952	0.62995	0.456765
1.5	1.907349	8.368857	0.946447	0.504231
2	1.490116	8.991815	1.263881	0.502576

3. Result & Conclusion

1. 실행 결과





3. Result & Conclusion

1. 실행 결과

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More Accurate

4th RK				
x	y	z	Error x(%)	Error y(%)
0	4	6	0	0
0.1	3.804918	6.17836	7.94E-06	0.000962
0.2	3.61935	6.353294	9.06E-06	0.001387
0.3	3.442832	6.524813	2.74E-06	0.00135
0.4	3.274923	6.692935	-3.8E-07	0.00096
0.5	3.115203	6.857678	-4.2E-06	0.000256
0.6	2.963273	7.019065	3.96E-06	-0.0007
0.7	2.818752	7.177124	-1.3E-05	-0.00185
0.8	2.68128	7.331881	-6.9E-06	-0.00317
0.9	2.550513	7.48337	1.54E-05	-0.0046
1	2.426123	7.631623	1.49E-05	-0.00614
1.1	2.307799	7.776676	-1E-05	-0.00774
1.2	2.195247	7.918566	2.08E-05	-0.00941
1.3	2.088183	8.057334	-5.1E-06	-0.01109
1.4	1.986341	8.193018	-1.1E-05	-0.0128
1.5	1.889466	8.325663	-1.1E-05	-0.0145
1.6	1.797316	8.455309	7.99E-06	-0.01621
1.7	1.70966	8.582003	1.59E-05	-0.01788
1.8	1.626279	8.705788	2.22E-05	-0.01953
1.9	1.546964	8.82671	-6.1E-06	-0.02116

4. Conclusion

1) 4^{th} RK가 더 좋은 결과를 보여주었으며, Heun 또한 꽤 좋은 결과를 보여주었다.
2) 작은 구간일 수록 더 적은 오차를 보여줌을 알 수 있었다.